On-shell recursion for string theory amplitudes on the disc and sphere

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based on: R.B., Daniele Marmiroli and Niels Obers

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scattering amplitudes are interesting.

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  - see also talks by [Henn] and [Broedel]
- what about 'strings'? Just in a flat background?
  - see also talk by [Mafra]

Fields vs strings state-of-the-art

field theory

- all Yang-Mills, gravity tree amplitudes in $D = 4$
- all 1-loop (massless) $\mathcal{N} = 1$ amplitudes
- all order conjectures in $\mathcal{N} = 4$

strings in flat background

- 6 point amplitude at tree level [Stieberger-Oprisa, 02]
- all multiplicity α'^2, α'^3 corrections to super Yang-Mills
- four point from effective action

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unacceptable, strings have much more symmetry

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- analytic progress based on 'analytic S-matrix'

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Main idea of our work

analytic S-matrix program (sixties)

construct scattering amplitudes from their physical singularities

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- revival inspired by [Witten, 03]
- exporting new QFT techniques to string theory natural (cf. [Stieberger-Taylor, 06-])
- this talk: on-shell recursion [Britto-Cachazo-Feng-(Witten), 04,05]

On-shell recursion relations

new input since 60s

if there is no nice complex parameter to play with: introduce one

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- amplitude $A \rightarrow A(z)$

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- finite z residues: lower point amplitudes $\rightarrow$ recursion!
- string amplitudes as infinite sums over three point amplitudes
- $z \rightarrow \infty$ related to UV ($\sim$ Regge) behavior
- different possible shifts related by crossing symmetry

4 point example [RB-Larsen-Obers-Vonk,08]

Veneziano amplitude

$$A_4 = A_4^{\text{part}}(s,t)\,(\text{Tr})_1 + A_4^{\text{part}}(t,u)\,(\text{Tr})_2 + A_4^{\text{part}}(u,s)\,(\text{Tr})_3$$

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- $\rightarrow$ all four point amplitudes

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- direct integral proof for adjacent shifts
- non-adjacent shifts through monodromy relations [Plahte, 70]

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all tree level amplitudes in a flat background obey on-shell recursion in any string theory, depending on kinematic invariant

- proven for open string / argued for closed string (see later)
- conjecture: **?** universal for shifted particles

CFT point of view

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adjacent shifts from CFT argument (based on [Brower et.al., 06])

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- Taylor expand exponential in y, assume $yz \sim 1$ and integrate

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- shifting gluon legs in the bosonic string

$$A_n(z) \sim \hat{\zeta}_1^{\mu} \hat{\zeta}_2^{\nu} \left(\frac{1}{z}\right)^{\alpha'(p_1+p_2)^2} \left(z \left[\eta_{\mu\nu} + \alpha' A_{\mu\nu}\right] + B_{\mu\nu} + \mathcal{O}\left(\frac{1}{z}\right)\right)$$

- analysis reduces to [Arkani-Hamed, Kaplan, 08]
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$$A(1,2,\ldots,n) + R_{12} A(2,1,3,\ldots,n) + R_{12} R_{23} A(2,3,1,\ldots,n) + \ldots = 0$$

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- general CFT: R_{12} obeys the Yang-Baxter equation

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  - is there a string amplitude bootstrap?