Yet more loop operators for $\mathcal{N} = 6$ super Chern-Simons-matter theory

Nadav Drukker
Humboldt Universität zu Berlin

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Based on `arXiv:0912.3006`: N.D and D. Trancanelli
`arXiv:0909.4559`: A. Kapustin, B. Willett, I. Yaakov
`arXiv:0912.3974`: M. Mariño, P. Putrov

Introduction and motivation

- The AdS/CFT correspondence is a powerful tool of modern theoretical physics.
- Allows to calculate gauge theory quantities at strong coupling (for large N).
- In the other direction, allows to calculate string theory quantities at large α'.
- In very simple situations the results agree, indicating that there is a non-renormalization principle at work.

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- In very simple situations the results agree, indicating that there is a non-renormalization principle at work.
- In other very specific cases one can derive (or guess) non-trivial functions that interpolate from weak to strong coupling:
  - BPS observables like Wilson loops, surface operators (topological subsectors).
  - Integrability: Cusp anomalous dimension. Konishi? Scattering amplitudes?
- This has been achieved so far only for the simplest example of exact AdS/CFT duality:

$$\mathcal{N} = 4 \text{ SYM} \qquad \Longleftrightarrow \qquad \text{Type IIB on } AdS_5 \times S^5$$

$\mathcal{N} = 6$ super Chern-Simons-matter theory

[Aharony, Bergman, Jafferis, Maldacena]

- Spring 2008: Building on Bagger-Lambert-Gustavsson, a new proposal for an exact AdS/CFT duality

$$d = 3, \quad \mathcal{N} = 6 \text{ SCS} \qquad \Longleftrightarrow \qquad \begin{cases} \text{M-thoery on } AdS_4 \times S^7/\mathbb{Z}_k \\ \text{Type IIA on } AdS_4 \times \mathbb{CP}^3 \end{cases}$$

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$$E(p)=\sqrt{J^2+4h^2(\lambda)\sin^2\tfrac{p}{2}}-J$$

  - Form obeyed at weak and strong coupling (constrained by symmetry).
  - In $\mathcal{N}=4$ SYM same structure with $h^2(\lambda)=\lambda/4\pi^2$.
  - In ABJM: Unknown function

$$h^2(\lambda)=\begin{cases} \lambda^2-4\lambda^4(4-\zeta(2))+\cdots & \text{small } \lambda \\ \frac{1}{2}\lambda+\cdots & \text{large } \lambda \end{cases}$$

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- Can a BPS Wilson loop be calculated exactly?

Outline

- Introduction and motivation.
- Review: $1/2$ BPS Wilson loop in 4d.
- $1/6$ BPS Wilson loop of ABJM.
- $1/2$ BPS Wilson loop of ABJM.
- Localization of $1/6$ BPS Wilson loop.
- Application to $1/2$ BPS Wilson loop.
- The super matrix model.
- Exact interpolating function.
- Summary.

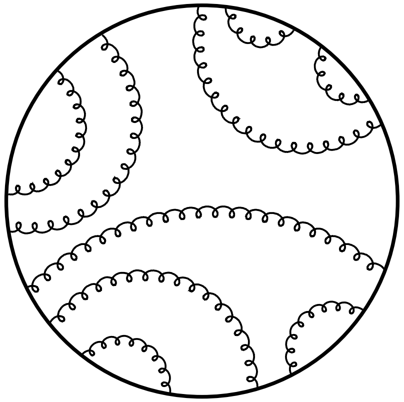
1/2 BPS Wilson loop in 4d

[Erickson, Semenoff, Zarembo][N.D, Gross]

- In $\mathcal{N}=4$ SYM can take the circular Wilson loop

$$W=\frac{1}{N}\operatorname{Tr}\mathcal{P}\exp\left[i\int(A_\mu\dot{x}^\mu+i\Phi|\dot{x}|)dt\right]$$

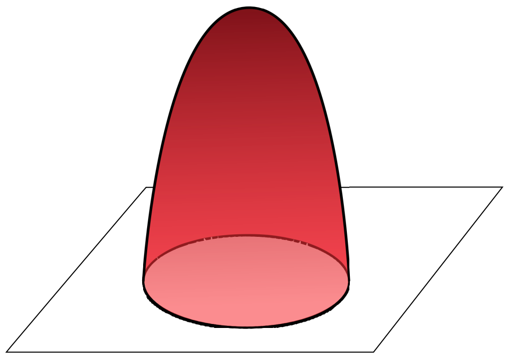
- Sum over ladder graphs given by a Gaussian matrix model!



$$\langle W\rangle=\frac{1}{Z}\int\mathcal{D}M\,\frac{1}{N}\operatorname{Tr}e^{M}\,e^{-\frac{2}{g^2}\operatorname{Tr}M^2}$$

- Proven to be exact. [Pestun]
- Generalized to theories with $\mathcal{N}=2$ supersymmetry.
- At large N and large g^2N this becomes

$$\langle W\rangle\quad\underset{N\to\infty}{\longrightarrow}\quad\frac{2}{\sqrt{g^2N}}I_1\left(\sqrt{g^2N}\right)\quad\underset{g^2N\to\infty}{\longrightarrow}\quad e^{\sqrt{g^2N}}$$



- Exactly matches the action for the corresponding classical string.
- By using D3 or D5 brane can also match $1/N$ terms.

Lightning review of ABJ(M) theory

- $U(N) \times U(M)$ gauge symmetry.
- Chern-Simons terms at levels k and $-k$.
- Kinetic terms for scalars and fermions.
- Very specific sextic scalar potential and $(C)^2(\psi)^2$ terms.
- Normally 3d super Chern-Simons has $\mathcal{N}=2$ or $\mathcal{N}=3$ SUSY.

Field content		dim	rep	
A_μ	gauge field	1	adj	1
$\hat{A}_\mu$	gauge field	1	1	adj
C_I	scalar	1/2	N	$\overline{M}$
$\bar{C}^I$	scalar	1/2	$\overline{N}$	M
ψ_I	fermion	1	$\overline{N}$	M
$\bar{\psi}_I$	fermion	1	N	$\overline{M}$

- This special quiver construction allows for $\mathcal{N}=6$ SUSY.
- For $k=1,2$ should be enhanced to $\mathcal{N}=8$ SUSY.

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- This special quiver construction allows for $\mathcal{N} = 6$ SUSY.
- For $k = 1, 2$ should be enhanced to $\mathcal{N} = 8$ SUSY.
- Is the low energy theory of N M2-branes on a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold (with $M - N$ fractional branes).
- Gravity dual: M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$.
- For $k^5 \gg N$ a better description is IIA on $AdS_4 \times \mathbb{CP}^3$.
- Analog of 't Hooft coupling: $\lambda = N/k$.

1/6 BPS Wilson loop of ABJM

$\begin{bmatrix}\text{N.D, Plefka}\\ \text{Young}\end{bmatrix}\begin{bmatrix}\text{Chen}\\ \text{Wu}\end{bmatrix}\begin{bmatrix}\text{Rey, Suyama}\\ \text{Yamaguchi}\end{bmatrix}$

- Borrowing from the 4d theory, to make a BPS loop we can add a scalar piece to the connection

$$A_\mu \dot{x}^\mu \qquad \to \qquad \mathcal{A} = A_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}|M_J^I C_I \bar{C}^J$$

- It's a bilinear on dimensional grounds and so it's in adjoint of $U(N)$.
- Checking SUSY gives unique solution

$$\delta_{\text{SUSY}}\mathcal{A} = 0 \qquad \Longrightarrow \qquad M_J^I = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Preserves two Poincaré supercharges and two superconformal ones $\Rightarrow$ $1/6$ BPS.
- Was calculated perturbatively to order λ^2

$$\langle W \rangle = 1 + \frac{5\pi^2}{6}\lambda^2 + \cdots$$

- There was no simple guess on how to extend to all orders.

Not satisfying....

- Such a Wilson loop is good supersymmetric observable.

  Yet:

  1. The fundamental string in AdS_4 ending on a circle at the boundary is $1/2$ BPS. It has action $S = -\pi\sqrt{2\lambda}$, so the VEV of the Wilson loop is

  $$\langle W \rangle_{\text{Large N}} \sim e^{\pi\sqrt{2\lambda}}$$

  2. The fundamental string preserves $SU(3) \subset SU(4)$ flavor symmetry. The Wilson loop $SU(2) \times SU(2)$.

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- This Wilson loop exists in any $\mathcal{N} = 2$ super Chern-Simons theory. [Gaiotto Yin]

- It does not see any SUSY enhancement when going form $\mathcal{N} = 2$ to $\mathcal{N} = 6$

- It can exist in either of the two groups or in both. Does not know of the special quiver construction of ABJM!

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Still an interesting observable!

1/2 BPS Wilson loop of ABJM

[Initiated in discussions with V. Niarchos, G. Michalogiorgakis]

- Such a Wilson loop in both gauge groups can be written in terms of an $(N+M)\times(N+M)$ connection

$$L=\begin{pmatrix}A_\mu\dot x^\mu+\frac{2\pi}{k}|\dot x|M_J^I C_I\bar C^J & 0\\ 0 & \widehat A_\mu\dot x^\mu+\frac{2\pi}{k}|\dot x|\widehat{M}_J^I\bar C^J C_I\end{pmatrix}$$

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- Generalization: Write an $(N+M)\times(N+M)$ superconnection

$$L=\begin{pmatrix} A_\mu\dot{x}^\mu+\frac{2\pi}{k}|\dot{x}|M_J^I C_I\bar{C}^J & \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\eta_I^\alpha\bar{\psi}_\alpha^I \\ \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\psi_I^\alpha\bar{\eta}_\alpha^I & \widehat{A}_\mu\dot{x}^\mu+\frac{2\pi}{k}|\dot{x}|\widehat{M}_J^I\bar{C}^J C_I \end{pmatrix}$$

The natural Wilson loop is then

$$W_{\mathcal{R}}\equiv\mathrm{Tr}_{\mathcal{R}}\mathcal{P}\exp\left(i\int L\,d\tau\right)$$

$\mathcal{R}$ is a representation of the supergroup $SU(N|M)$.

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Can this be supersymmetric?

Checking SUSY

- To preserve $SU(3)$ R-symmetry we choose

$$M_J^I = \widehat{M}_J^I = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Under half of the supercharges

$$\delta\mathcal{A} \propto C\psi_1^+ + \bar{\psi}_+^1 \bar{C} \qquad \delta\widehat{\mathcal{A}} \propto \psi_1^+ C + \bar{C}\bar{\psi}_+^1$$

- Chose therefore

$$\eta_I^\alpha \propto \delta_I^1 \delta_+^\alpha$$

$$L = \begin{pmatrix} \mathcal{A} & \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\eta_I^\alpha \bar{\psi}_\alpha^I \\ \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\psi_I^\alpha \bar{\eta}_\alpha^I & \widehat{\mathcal{A}} \end{pmatrix}$$

For the time-like line:

$$\mathcal{A} = A_0 + \frac{2\pi}{k} M_J^I C_I \bar{C}^J$$

$$\widehat{\mathcal{A}} = \widehat{A}_0 + \frac{2\pi}{k} M_J^I \bar{C}^J C_I$$

- The variation of the fermion under the SUSY

$$\delta\psi_1^+ = 2\gamma^\mu D_\mu \bar{C} + \bar{C}C\bar{C} \qquad D_\mu \bar{C} = \partial_\mu \bar{C} + i\widehat{A}_\mu \bar{C} - i\bar{C}A_\mu$$

- For a choice of half the supercharge with specific chiralities we get $(\gamma^\mu)_+^+ = \delta_0^\mu$. All the cubic terms in C organize such that

$$\delta\psi_1^+ = 2\mathcal{D}\bar{C} \qquad \mathcal{D}\bar{C} = \partial_0 \bar{C} + i\widehat{\mathcal{A}}\bar{C} - i\bar{C}\mathcal{A}$$

  A covariant derivative along the line with the modified connection!

- We finally get that under the three supercharges parameterized by $\bar\theta^{1I}_+$ and the three by $\bar\theta^{IJ+}$

$$\delta L = \frac{8\pi}{k}\bar\theta^{1I}_+\begin{pmatrix} C_I\,\psi_1^+ & \sqrt{\frac{k}{8\pi}}\eta\mathcal{D}_0 C_I \\ 0 & \psi_1^+\,C_I \end{pmatrix} - \frac{4\pi}{k}\varepsilon_{1IJK}\,\bar\theta^{IJ+}\begin{pmatrix} \bar\psi_+^1\,\bar C^K & 0 \\ \sqrt{\frac{k}{8\pi}}\bar\eta\mathcal{D}_0\bar C^K & \bar C^K\,\bar\psi_+^1 \end{pmatrix}$$

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  This is not zero...

- But the variation of the entire Wilson loop is.

- Crucial point: Expanding the Wilson loop in a power series in the fermionic terms it is possible to integrate the modified covariant derivative!

$$W_{\mathcal{R}}=\mathrm{Tr}_{\mathcal{R}}\mathcal{P}\left[e^{i\int L_B\,d\tau}\left(1+i\int_{-\infty}^{\infty}d\tau_1\,L_F(\tau_1)-\int_{-\infty}^{\infty}d\tau_1\int_{\tau_1}^{\infty}d\tau_2\,L_F(\tau_1)L_F(\tau_2)+\dots\right)\right]$$

- The variation of the linear term is a total derivative.

- The variation of $e^{i\int L_B}$ gives at the linear order $\delta\mathcal{A}\sim C\psi$. This cancels the variation of the quadratic term in L_B, after integration by parts (for $\eta\bar\eta=2i$).

- This repeats at all orders: Integrating by parts the total derivative is canceled by an insertion of $\int\delta L_B$ into the term with two fewer L_F factors.

Localization of $1/6$ BPS Wilson loop

[Kapustin, Willett, Yaakov]

Though it's not $1/2$ BPS or dual to the simplest fundamental string (Or M2 brane), can one get an exact interpolating function for the $1/6$ BPS Wilson loop?

- Consider any $\mathcal{N} = 2$ super Chern-Simons matter theory on S^3.
- Take a Wilson loop of that theory on the equator invariant under a supercharge Q.
- Add to the action a Q-exact term of the form $t\, Q(\Psi Q \Psi)$.
- VEV of Q-invariant observables is unmodified by this insertion.
- Take t large and look at the saddle points of $(Q\Psi)^2$.
- Get the VEV of the Wilson loop from the classical value at the saddle point and the one loop determinant around that point.

- For a theory with one $U(N)$ vector multiplet

$$Z=\int\prod_{a=1}^{N}d\lambda_a\,e^{ik\pi\lambda_a^2}\prod_{a<b}\sinh^2(\pi(\lambda_a-\lambda_b))$$

- Wilson loop in the fundamental: Insert into the integral

$$\sum_{a=1}^{N}e^{2\pi\lambda_a}$$

- This is the matrix model for regular $U(N)$ topological Chern-Simons on S^3.

[Mariño] [Aganagic, Klemm Mariño, C. Vafa] [Halmagyi Yasnov]

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- Applying this to ABJ(M) theory one finds the partition function

$$Z = \int \prod_{a=1}^{N} d\lambda_a \, e^{ik\pi\lambda_a^2} \prod_{\hat{a}=1}^{M} d\hat{\lambda}_{\hat{a}} \, e^{-ik\pi\hat{\lambda}_{\hat{a}}^2} \frac{\prod_{a<b} \sinh^2(\pi(\lambda_a - \lambda_b)) \prod_{\hat{a}<\hat{b}} \sinh^2(\pi(\hat{\lambda}_{\hat{a}} - \hat{\lambda}_{\hat{b}}))}{\prod_{a,\hat{a}} \cosh^2(\pi(\lambda_a - \hat{\lambda}_{\hat{a}}))}$$

- $1/6$ BPS Wilson loop in the fundamental of first group is same insertion as above.

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- $1/6$ BPS Wilson loop in the fundamental of first group is same insertion as above.
- Interpretation: This is the matrix model for $SU(N|M)$ Chern-Simons on $S^3/\mathbb{Z}_2$:
  1. The odd terms contribution to the Vandermonde determinant go in the denominator.
  2. The orbifold allows for two saddle points: $\hat\lambda$ shifted by $i/2$.
- This Wilson loop is not the most natural observable!

Localization for $1/2$ BPS Wilson loop

- Can use the same localization for the $1/2$ BPS loop (they share the supercharges).
- Take a $1/6$ BPS Wilson loop of the form ($\mathcal{R}$ is a rep of $SU(N|M)$)

$$W_{\mathcal{R}}^{(1/6)} \equiv \operatorname{Tr}_{\mathcal{R}} \mathcal{P} \exp\left(i \int L^{(1/6)}\, d\tau\right), \qquad L^{(1/6)} = \begin{pmatrix} \mathcal{A}^{(1/6)} & 0 \\ 0 & \widehat{\mathcal{A}}^{(1/6)} \end{pmatrix}$$

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- The Wilson loop in the fundamental of $SU(N|M)$ inserts into the matrix model

$$W = \sum_{a=1}^{N} e^{2\pi\lambda_a} + \sum_{\hat{a}=1}^{M} e^{2\pi\hat{\lambda}_{\hat{a}}}$$

- For a general representation the insertion is

$$W_{\mathcal{R}} = \mathrm{Tr}_{\mathcal{R}} \begin{pmatrix} \mathrm{diag}(e^{2\pi\lambda_a}) & 0 \\ 0 & \mathrm{diag}(e^{2\pi\hat{\lambda}_{\hat{a}}}) \end{pmatrix} = \mathrm{sTr}_{\mathcal{R}} \begin{pmatrix} \mathrm{diag}(e^{2\pi\lambda_a}) & 0 \\ 0 & -\,\mathrm{diag}(e^{2\pi\hat{\lambda}_{\hat{a}}}) \end{pmatrix}$$

- These are the natural observables in this super matrix model!

Exact interpolating function

[Mariño, Putrov]

- The matrix model for $SU(N+M)$ Chern-Simons on $S^3/\mathbb{Z}_2$ has a large-N solution.
- To get $SU(N|M)$ analytically continue $M \to -M$.
- To compare with string theory we should take λ large (with $M \sim N$).
- For the $1/2$ BPS loop the result is

$$\left\langle W^{(1/2)} \right\rangle = \frac{1}{8\pi\lambda} e^{\pi\sqrt{2\lambda}}$$

- For the $1/6$ BPS loop the result is

$$\left\langle W^{(1/6)} \right\rangle = \frac{1}{2\pi\sqrt{2\lambda}} e^{\pi\sqrt{2\lambda}}$$

- The exponent precisely matches the action of a fundamental string!
- Different prefactors could be related to localized/smeared string.

Summary

- The BPS Wilson loop provide the first weak to strong coupling interpolating function in ABJM theory.
- The 1/2 BPS is the natural dual of the fundamental string in AdS_4.
  - Has a very natural expression in the supergroup Chern-Simons matrix model.
- 1/6 BPS loop can also be calculated exactly.
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  – Other exactly calculable quantities — “AGT for 3d theories”?
- ABJM theory is harder than $\mathcal{N} = 4$ SYM, but not impossible!

The end