Scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills

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based on

F. Alday, J. H., J. Plefka and T. Schuster, arXiv:0908.0684 [hep-th]
J. H., S. Naculich, H. Schnitzer and M. Spradlin arXiv:1001.1358 [hep-th]

Nordic Strings, Hannover, Feb 23rd, 2010

Scattering amplitudes: Nice surprises in $\mathcal{N} = 4$ SYM

Can we hope to determine all amplitudes, i.e. for arbitrary number of legs and loops?

1. symmetries
   $\Rightarrow$ amplitudes in $\mathcal{N} = 4$ possess a hidden dual conformal symmetry
   [Drummond, J. H., Korchemsky, Sokatchev]
2. tree-level: all $\mathcal{N} = 4$ amplitudes are known [Drummond, J. H.]
3. loop-level: all-loop BDS ansatz for MHV amplitudes
   [Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov]
   $\Rightarrow$ correct for 4 and 5 particles if dual conformal symmetry holds
   [Drummond, J. H., Korchemsky, Soktachev]
   (known to be modified for 6 particles)
4. new duality between MHV amplitudes and Wilson loops
   [Alday, Maldacena; Drummond, J. H., Korchemsky, Soktachev; Brandhuber, Heslop, Travaglini]
5. AdS/CFT: computations of amplitudes at $\lambda \to \infty$
   [Alday, Maldacena; Alday, Gaiotto, Maldacena; Alday, Maldacena, Sever, Vieira]
6. revival of twistor dualities!? [Arkani-Hamed et al.; Mason, Skinner; Spradlin et al, ...]

Scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills

- Symmetries
- Scattering on the Coulomb branch of $\mathcal{N} = 4$ SYM
  - Extended dual conformal symmetry
  - Exponentiation
  - Regge limit

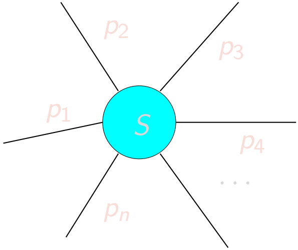
Tree-level symmetries

What are the symmetries? How are they realized?

1. superconformal symmetry (follows from action) [Witten]

$$k^{\alpha\dot\alpha} = \sum_i p_i^{\beta\dot\beta} \frac{\partial}{\partial p_i^{\beta\dot\alpha}} \frac{\partial}{\partial p_i^{\alpha\dot\beta}} + \ldots$$

$$k^{\alpha\dot\alpha} A = 0$$



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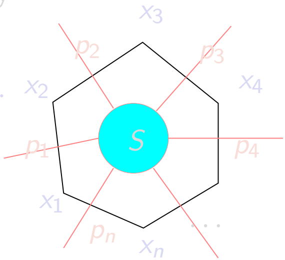

$$p_i^\mu = x_i^\mu - x_{i+1}^\mu$$

What is the closure of the two symmetry algebras?

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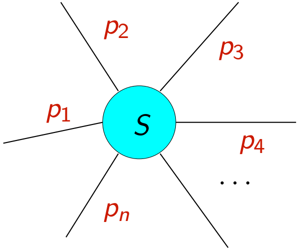

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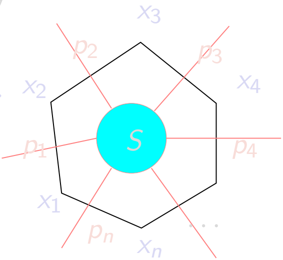

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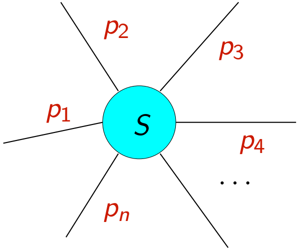

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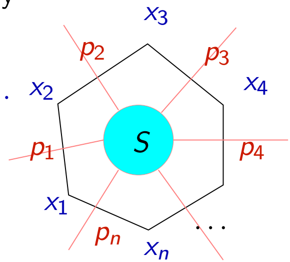
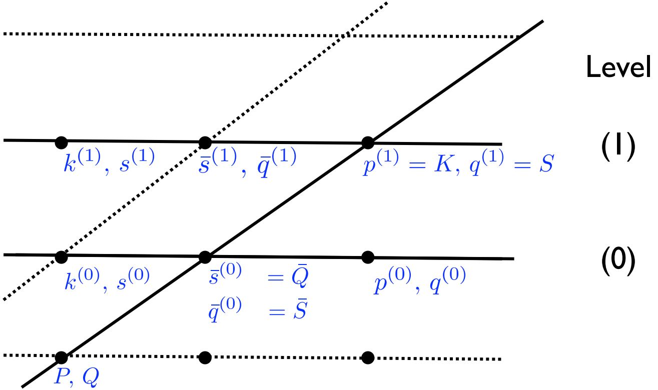


$$\boxed{p_i^\mu = x_i^\mu - x_{i+1}^\mu}$$

What is the closure of the two symmetry algebras?

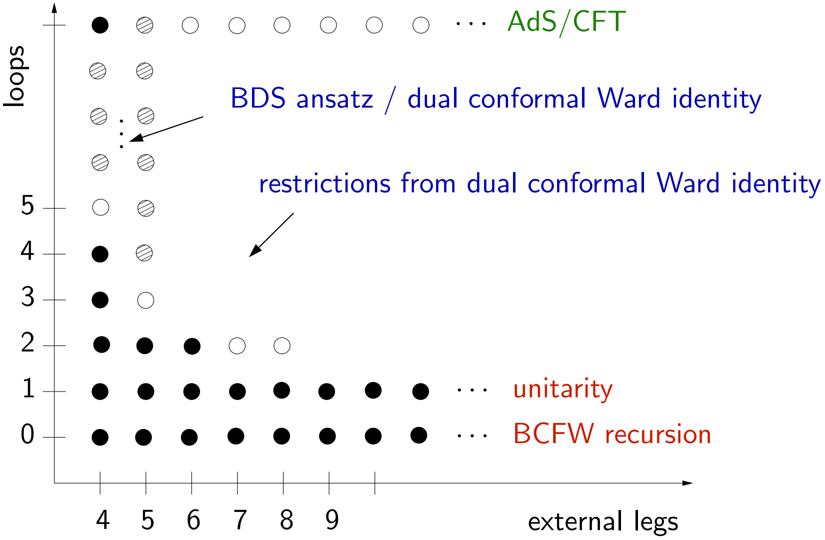
Summary of Yangian structure

- Combination of standard and dual superconformal symmetry gives Yangian $Y[\mathfrak{psu}(2,2|4)]$ [Drummond, J. H., Plefka] [Picture: Beisert]



- Tree level superamplitudes invariant: $\boxed{\mathcal{J} \circ \mathbb{A}_n^{\text{tree}} = 0}$ for $\mathcal{J} \in Y[\mathfrak{psu}(2,2|4)]$.
- string theory interpretation: fermionic T-duality [Berkovits, Maldanca; Beisert, Ricci, Tseytlin, Wolf]

Summary of current understanding

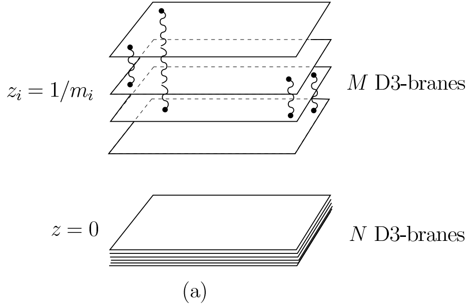


- Diagram has three important ingredients:
  analytic properties, symmetries (+IR structure), AdS/CFT

An alternative regularization [Alday, Henn, Plefka, Schuster]

- string picture:

Alday, Maldacena



- bosonic + fermionic T-duality is relevant [Alday, Maldacena; Berkovits, Maldacena]
- isometries of AdS_5 in T-dual theory

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$

$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$

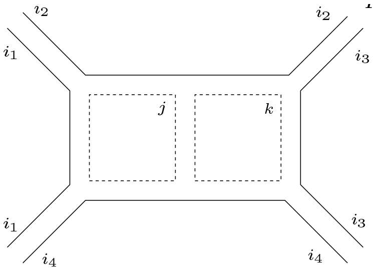
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- should correspond to Higgs mechanism in the field theory
- Expectation: Amplitudes regulated by Higgs masses should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$!

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

[Alday, Henn, Plefka, Schuster]

- $U(N+M) \to U(N) \times U(1)^M$
  - 'light' $(m_i - m_j)$ fields $\to$ zero mass for $m_i = m$
  - 'heavy' m_i fields $\to$ mass m for $m_i = m$



- scatter fields with M, M indices,
  only allow loops in N-part of $U(N+M)$
  $N \gg M$
  $\to$ renders amplitudes IR finite

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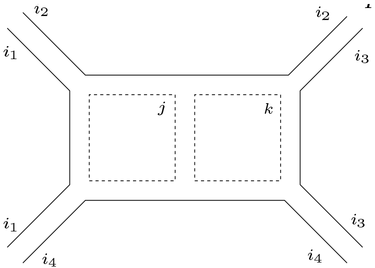

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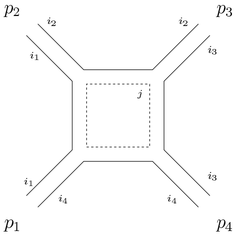
One loop test of extended dual conformal symmetry

- Consider the purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\Big(1 + \lambda\, I^{(1)}(s,t,m_i) + O(a^2)\Big)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables ($p_i = x_i - x_{i+1}$)



$$= \int d^4 x_a \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{1a}^2 + m_1^2)(x_{2a}^2 + m_2^2)(x_{3a}^2 + m_3^2)(x_{4a}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\qquad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5 \hat{x}_a \frac{\delta(\hat{x}_a^{M=4})}{\hat{x}_{1a}^2 \hat{x}_{2a}^2 \hat{x}_{3a}^2 \hat{x}_{4a}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

Exponentiation in Higgs regularization

- reminder: dimensional regularization [Bern, Dixon, Smirnov]

$$\log M_4 = \sum a^\ell \left[-\frac{\Gamma_{\text{cusp}}^{(\ell)}}{2(\ell\epsilon)^2} - \frac{\mathcal{G}_0^{(\ell)}}{2\ell\epsilon} \right] \left[\left(\frac{\mu^2}{s}\right)^{\ell\epsilon} + \left(\frac{\mu^2}{t}\right)^{\ell\epsilon} \right] + \frac{1}{4}\Gamma_{\text{cusp}}(a) \left[\log^2\frac{s}{t} + \frac{4}{3}\pi^2 \right] + c(a) + O(\epsilon)$$

interference of $1/\epsilon$ and $O(\epsilon)$: $1/\epsilon \times O(\epsilon) = O(1)$
$\Rightarrow$ in order to compute $\log M$, need $O(\epsilon)$ terms in M

- analog of BDS in Higgs regularization: [Alday, J. H., Plefka, Schuster; J. H., Naculich, Schnitzer, Spradlin]

$$\log M_4 = -\frac{1}{4}\Gamma_{\text{cusp}}(a) \left[\log^2\frac{s}{m^2} + \log^2\frac{t}{m^2} \right] - \tilde{G}_0(a) \left[\log\frac{s}{m^2} + \log\frac{t}{m^2} \right] + \frac{1}{4}\Gamma_{\text{cusp}}(a) \left[\log^2\frac{s}{t} + \pi^2 \right] + \tilde{c}(a) + O(m^2)$$

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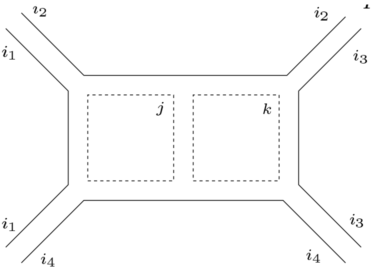
Extended dual conformal invariance at higher loops

- At 2 loops: Only one integral is allowed by extended dual conformal symmetry:



Similarly restricts possible integrals at higher loops.

[cf. also Drummond, Henn, Smirnov, Sokatchev '06]

- Computed this integral in $m_i \to 0$ limit using Mellin-Barnes techniques.
- No $\frac{1}{\epsilon} \times \epsilon \to 1$ 'interference' as in dimreg, here $\ln(m^2) \times m^2 \to 0$

Two- and three-loop exponentiation

- analog of BDS in Higgs regularization: [Alday, J. H., Plefka, Schuster; J. H., Naculich, Schnitzer, Spradlin]

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- verified by computing dual conformal integrals up to $O(m^2)$
  - at two loops [Alday, J. H., Plefka, Schuster]
  - at three loops [; J. H., Naculich, Schnitzer, Spradlin]

Regge limits for amplitudes on the Coulomb branch

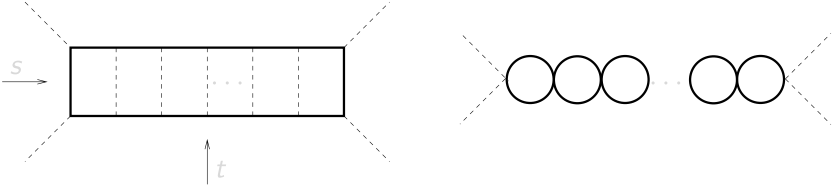
- take Regge limit $t = (p_2 + p_3)^2 \to \infty$ [J. H., Naculich, Schnitzer, Spradlin]

  expect

$$\beta(s/m^2)\left(\frac{t}{m^2}\right)^{\alpha(s/m^2)-1} + \mathcal{O}(m^2)$$

  trajectory $\alpha(s/m^2) - 1 = -\frac{1}{4}\gamma(a)\log(s/m^2) - \tilde{\mathcal{G}}_0(a)$

- determine leading Regge behavior [Eden et al, The analytic S-matrix]



- horizontal ladders give leading log (LL) contribution at L loops

$$\frac{(-1)^L}{L!}\log^L\left(\frac{t}{m^2}\right)K^L\left(\frac{s}{m^2}\right), \qquad K\left(\frac{s}{m^2}\right) = \log\left(\frac{s}{m^2}\right) + \mathcal{O}(m^2)$$

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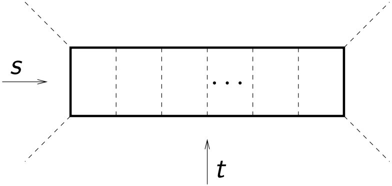

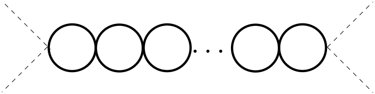

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Speculations about all loops and legs

- It seems reasonable to speculate that [J. H., Naculich, Schnitzer, Spradlin]
(similar conjecture for off-shell regulator: [Drummond, Korchemsky, Sokatchev])

$$M_n = 1 + \sum_{\mathcal{I}} a^{L(\mathcal{I})} \, c(\mathcal{I}) \, \mathcal{I} \, ,$$

where: coupling a, loop order $L(\mathcal{I})$
coefficients $c(\mathcal{I}) \Rightarrow$ compute by (generalized) unitarity
integrals $\mathcal{I} \Rightarrow$ restricted set of extended dual conformal integrals

- additional constraints from expected IR structure

$$M_n = \exp\left[-\frac{1}{8}\Gamma_{\text{cusp}}(a) \sum_i \log^2 \frac{s_i}{m^2} - \frac{1}{2}\tilde{G}_0(a) \sum_i \log \frac{s_i}{m^2} + \mathcal{O}(\log^0 m^2) \right]$$

- insights from analytic structure for generic m^2, and Regge limit(s)?
- further constraints from the (broken) conventional conformal symmetry?

Summary

- Higgs regulator for planar $\mathcal{N} = 4$ SYM
  - Higgs regulator makes dual conformal symmetry exact
  - restricts integral basis
  - exponentiated amplitude easier to compute
  - Regge limit: leading log computed to all orders!
- can we understand the all-loop structure for six points?
- can we learn more from string theory for perturbative gauge theory computations?