Thermodynamic instability of doubly spinning black objects

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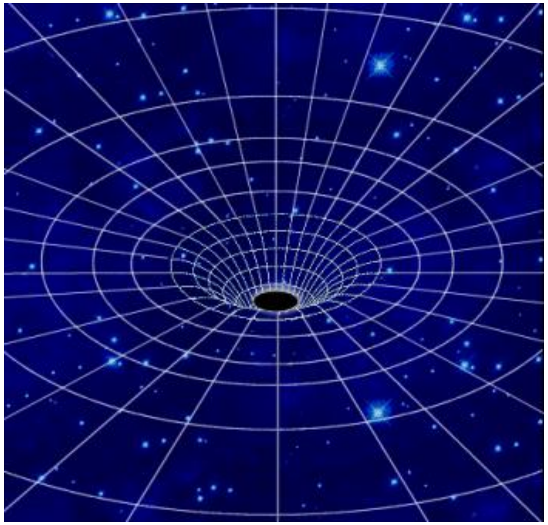
Dumitru Astefanesei, Robert B. Mann, MJR, Cristian Stelea   arXiv:0909.3852 [hep-th]
Dumitru Astefanesei, MJR, Stefan Theisen   arXiv:0909.0008 [hep-th]   & to appear

Outline

- Motivation
- Introduction
  - BH solutions in D-dim
  - Ultra-spinning BH
- Thermodynamics
  - Thermal ensembles
  - Grand-canonical ensemble
  - Thermodynamic stability
- Instabilities from Thermodyn.
  - Membrane phase
  - Critical points & turning points.
- Summary and outlook

Motivation

Black holes are the most elementary and fascinating objects in General Relativity

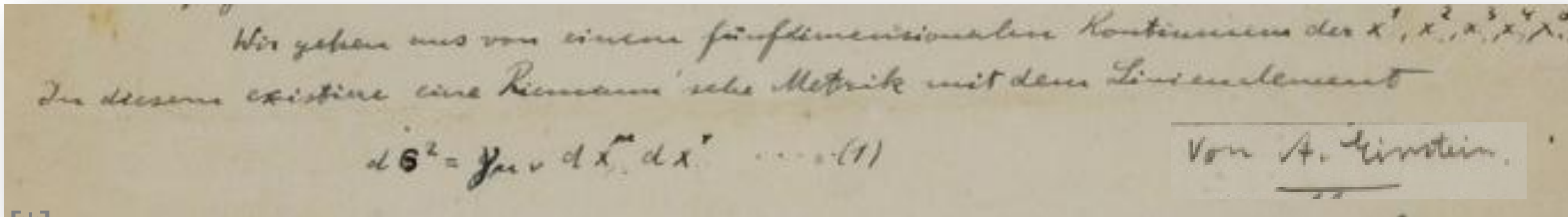


In the presence of black holes the effects of the space-time curvature are dramatic

The study of their properties is essential to understand better the dynamics of space-time

In string theory, mathematics and recent cutting edge experiments black objects are also relevant.

On the BH species (by means of natural selection)



[*]

Vacuum Einstein´s equation → $R_{\mu\nu}=0$ $\mu, \nu = 1,2,\ldots,D$

Boundary conditions → Asymptotically flat

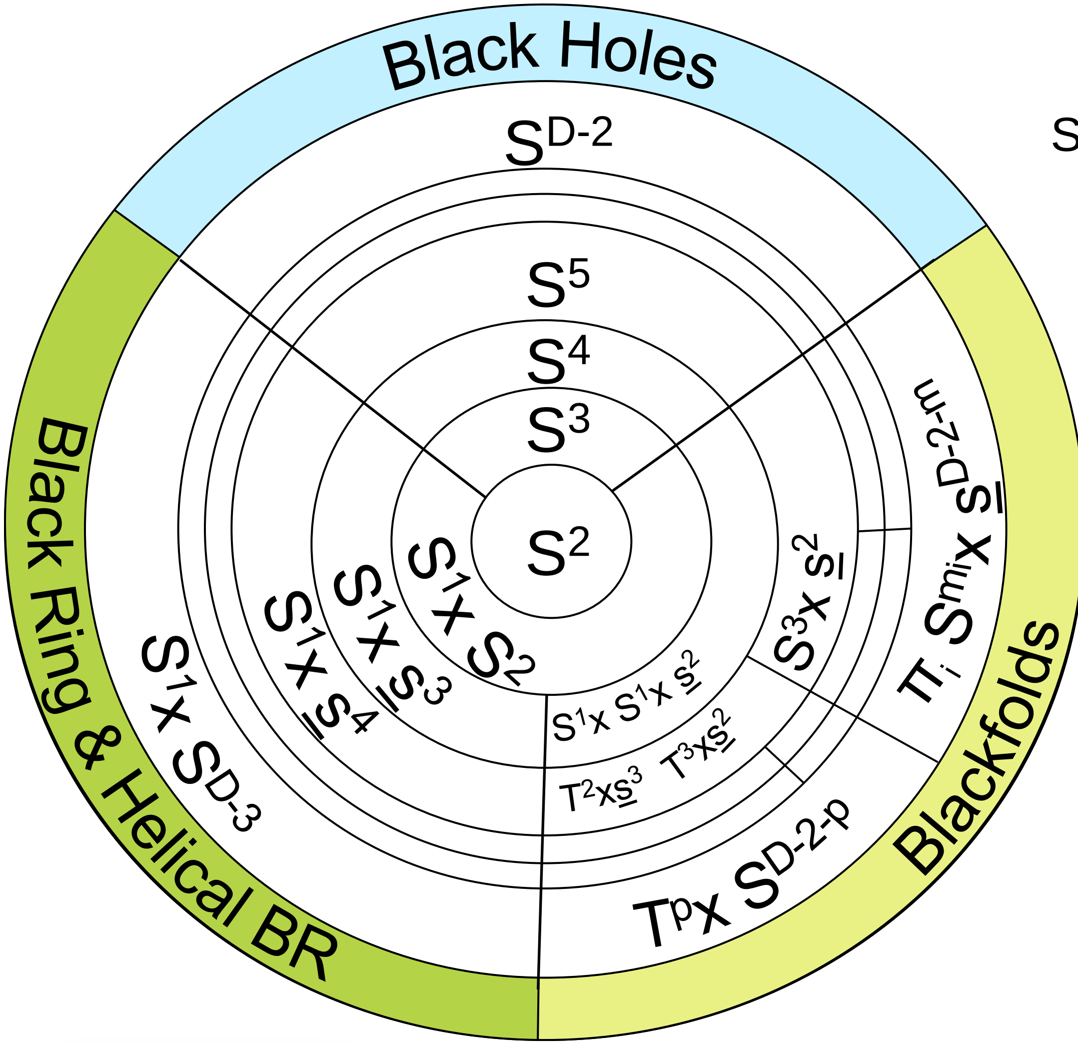
Equilibrium

Stationary – no time dependence

Regular solutions on and outside the event horizon

[*] We start from a five dimensional continuum which is x^1,x^2,x^3,x^4,x^0.
In it there exists a Riemannian metric with a line element $ds^2= g_{\mu\nu}\, dx^\mu dx^\nu$

On the BH species (by means of natural selection)



Goal:

Study dynamical properties of BH in D-dim to learn how the their solutions connect.

- Relies on the instabilities of the BH solutions
- Applications: Use to construct new solutions

$$D^i \tau_{ij} = 0$$

The boundary stress tensor satisfies a local conservation law

Jargon and remainder

On the number of angular momenta

Maximum # angular momenta: $j_1 \quad j_2 \quad . . . \quad j_{[(D-1)/2]}$

On the number of horizons

One compact horizons: uni horizon black hole solutions

Disconnected compact horizons: multi horizon black hole solutions

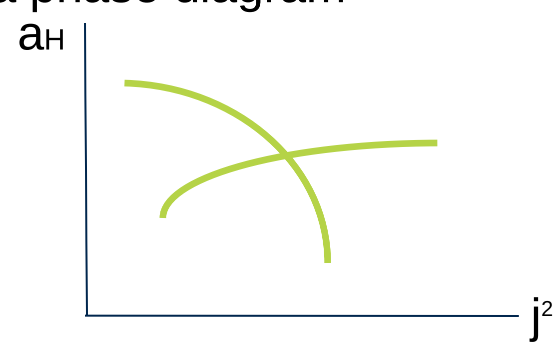
On how we compare solutions

To compare solutions we need to fix a common scale *Classical* GR

→ We'll fix the mass *M* equivalently and factor it out to get dimensionless quantities

a_H | j^2 | ω | …

Compare by drawing diagrams i.e. a phase diagram



Phase diagram of black objects

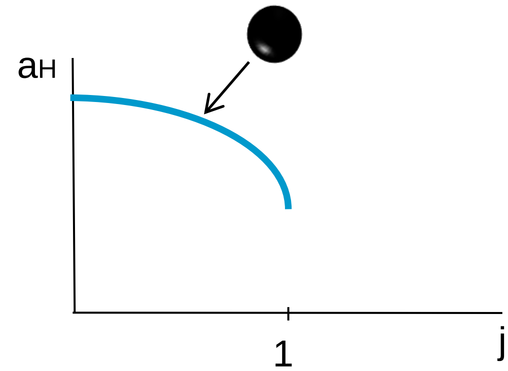
One&Two angular momenta + Vacuum + Asymtotically flat

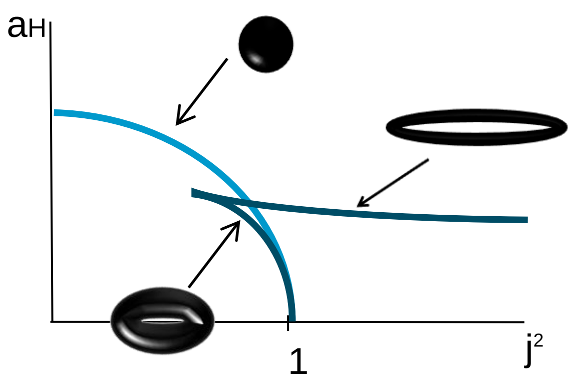
BH w/ one J in D-dim

What do we know about black objects?

In D=4 dimensions

-Kerr black hole





In D=5 dimensions

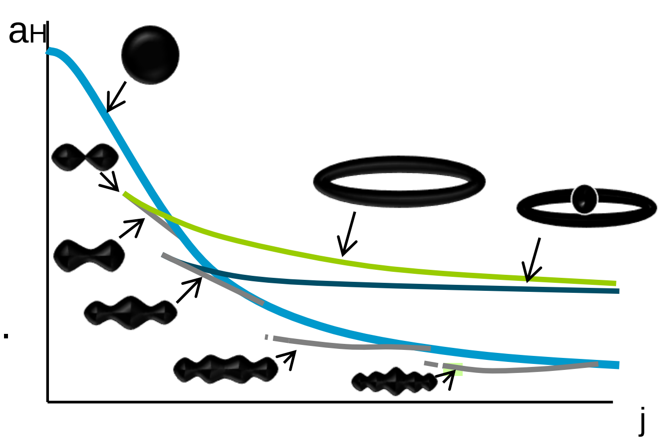
-the Myers-Perry black hole

-black ring

In D>5 dimensions

- the Myers-Perry black hole
- *thin* black ring and *black saturn*

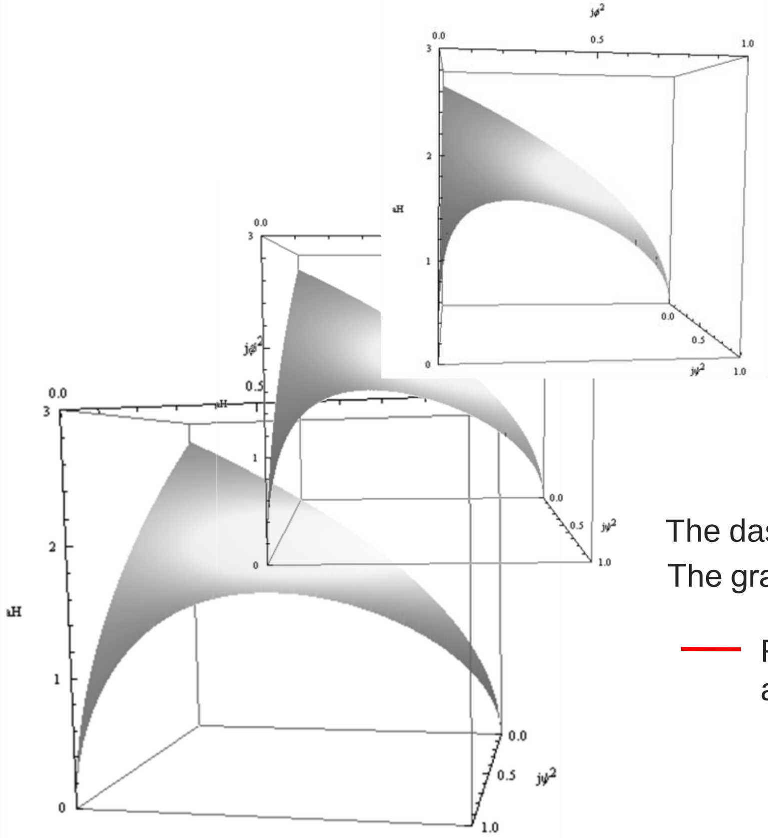
It seems that there is an infinite number of BHs.

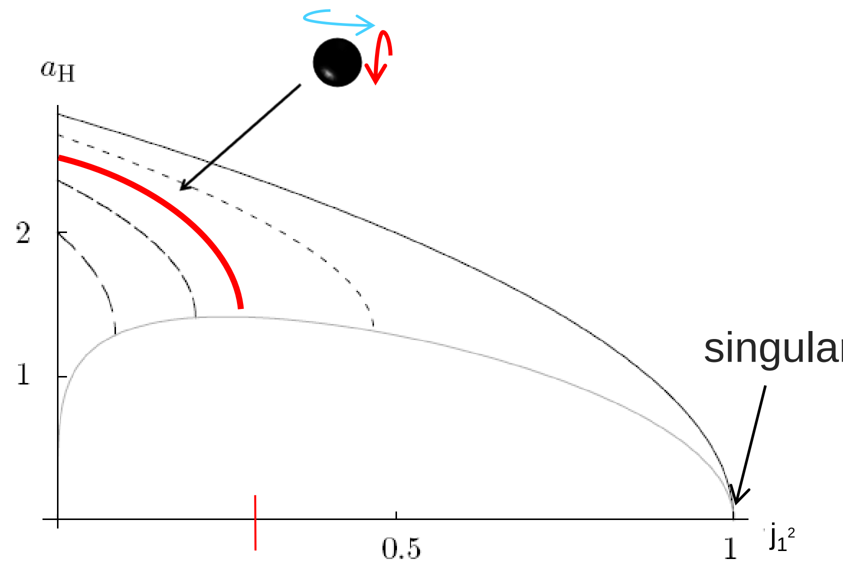


BH w/ two J in D-dim

The generalization of the black hole solution with ANY # angular momenta is the Myers-Perry (MP)solution.

D=5





The dashed lines show MP for fixed values of j_2 =0.1,0.3,0.5 right to left
The gray curve is the phase of zero temperature BH's

Representative phase of MP-BHs with one of the two angular momentafixed

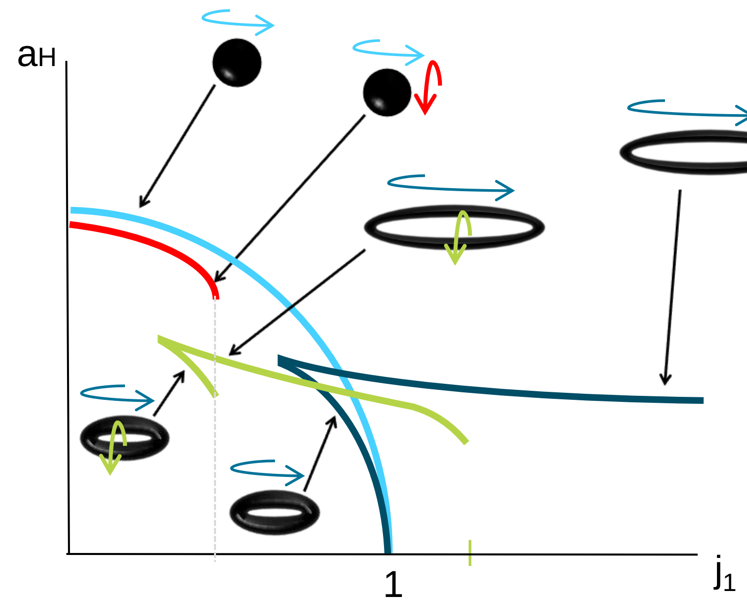
BH w/ two J in D-dim

What do we know these black objects?

In D=5 dimensions

- -Myers-Perry Black Hole (BH)
- -Black Ring (BR)
- -Helical BH
- -Black Saturn
- -Bicycling BR

Not shown here

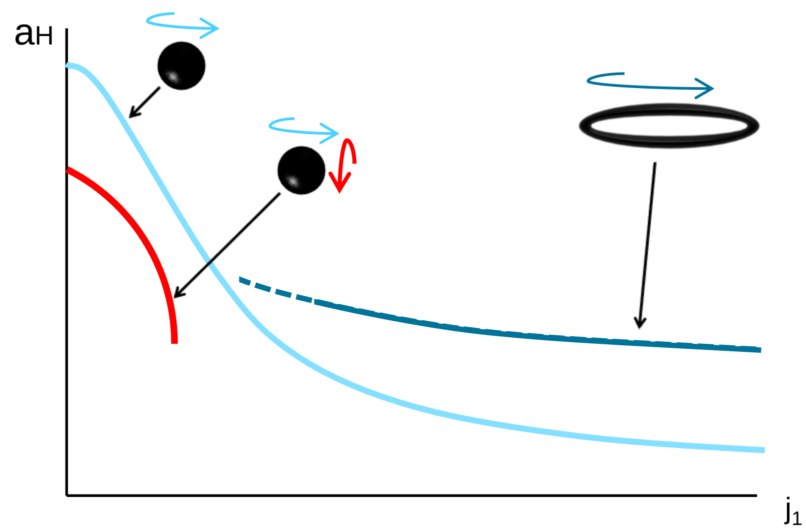


j_2 is fixed

In D>5 dimensions

- -Myers-Perry Black Hole
- -Black Ring (BR)
- -Helical BH
- -Black Saturn
- -Bicycling BR
- -Blackfolds

Not shown here



j_2 is fixed

BH w/ two J in D-dim

Why are we interested doubly spinning solutions?

To study the properties of these solutions
in particular since they have an extremal limit.

Asymptotically flat 

T=0

SUSY

Non SUSY

Are harder to find
since their interaction
play a BIG role

Black Holes with T=0 are interesting because they can teach us
about the microscopic origin of their physical properties

Ultra-spinning black objects

One&Two angular momenta + Vacuum + Asymtotically flat

Ultra-spinning black objects

$$ds^2 = -\frac{(\Delta - a^2\sin^2\theta)}{\Sigma}dt^2 - 2a\sin^2\theta\frac{(r^2+a^2-\Delta)}{\Sigma}dt\,d\phi + \left(\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta\, d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + r^2\cos^2\theta\, d\Omega^2_{D-4}.$$

$$ds^2 = -\frac{F(x)}{F(y)}\left(dt + R\sqrt{\lambda\nu}\,(1+y)\,d\psi\right)^2 + \frac{R^2}{(x-y)^2}\left[-F(x)\left(G(y)\,d\psi^2 + \frac{F(y)}{G(y)}dy^2\right) + F(y)^2\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right)\right] \quad (3.1)$$

$$\Sigma = r^2 + a^2\cos^2\theta$$
$$\Delta = r^2 - 2\,m\,r + a^2$$

$$r^2 + a^2 - 2mr^{5-D} = 0\,.$$

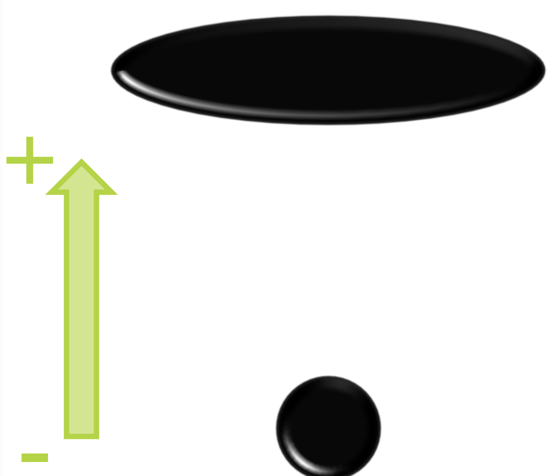
where $F(\xi) = 1 - \lambda\xi\,, \qquad G(\xi) = (1-\xi^2)(1-\nu\xi)\,.$

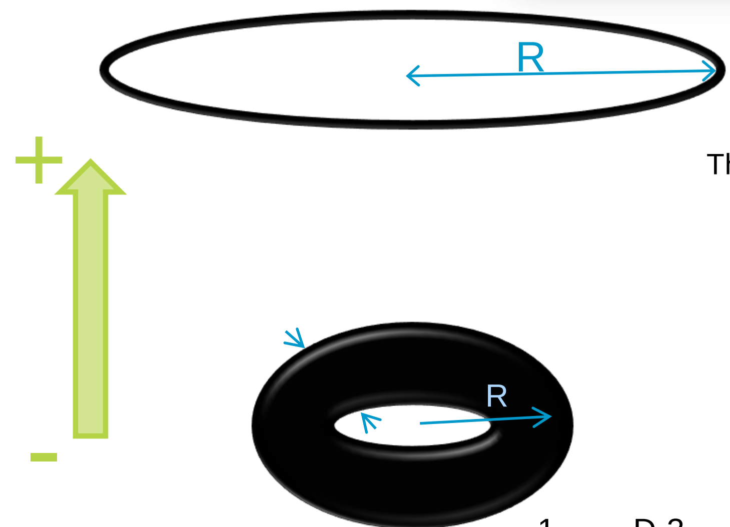
$$-1 \le x \le 1\,, \qquad -\infty < y \le -1\,, \quad \lambda^{-1} < y < \infty$$

Parameters in the solution $0 \le \nu < \lambda < 1\,.$

Balance condition

$$\lambda = \lambda_c \equiv \frac{2\nu}{1+\nu^2}$$





Ultra-spinning black objects

In certain regimes black holes and black rings behave like black strings and black p-branes.

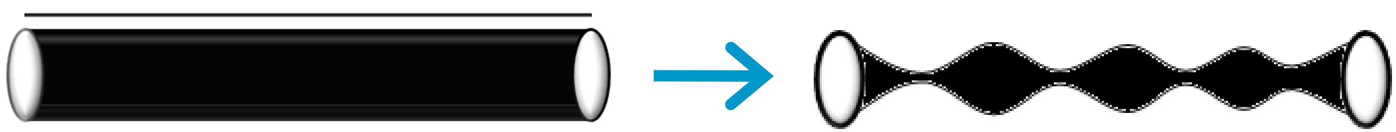
Emparan and Myers

Black strings and branes exhibit Gregory-Laflamme instability

Black Holes and black rings in ultra-spinning regime will inherit the instabilities.

A black hole solution which is thermally unstable in the grand-canonical ensemble will develop a classical instability.

Gubser and Mitra



Branch of static **lumpy black strings**

Gubser and Wiseman

At which value of j_m do the black objects start behaving like black strings/branes?

If black objects are thermally unstable for j_{th} does this imply that there they are classically unstable? Is there any relation with the j_m?

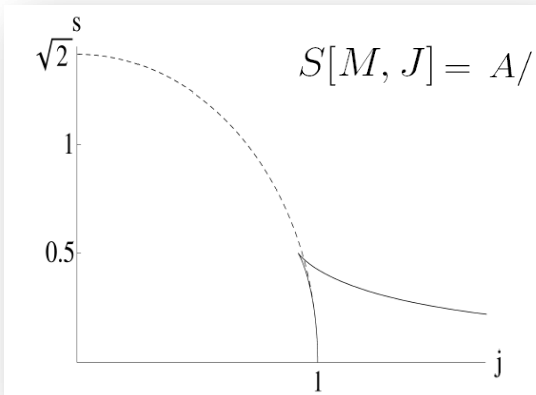
Thermodynamics of black objects

Thermal ensembles

Which ensemble is the most suitable for this analysis?

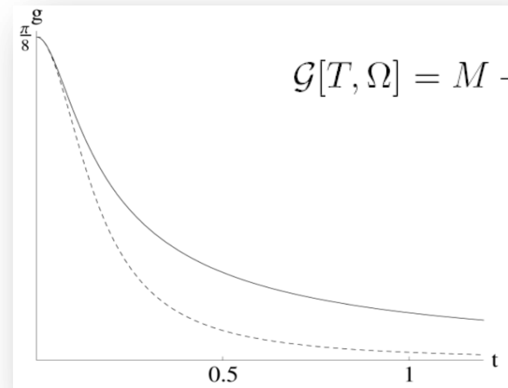
Entropy – microcanonical ensemble

$$S[M, J] = A/(4\,G)$$



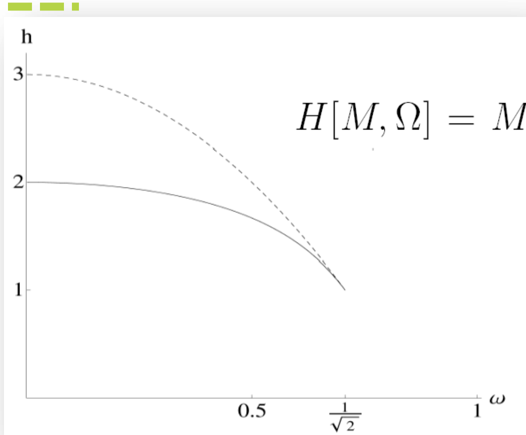
Gibbs potential – grand canonical ensemble

$$\mathcal{G}[T, \Omega] = M - \Omega\, J - T S,$$



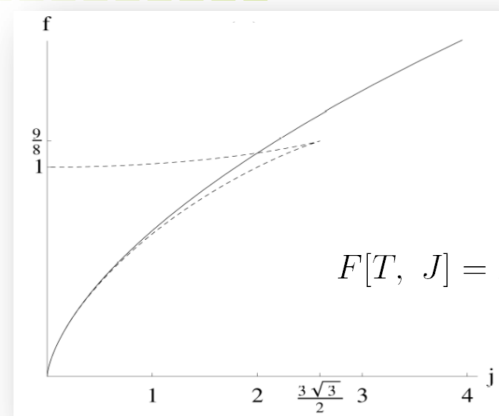
Enthalpy

$$H[M, \Omega] = M - \Omega\, J$$



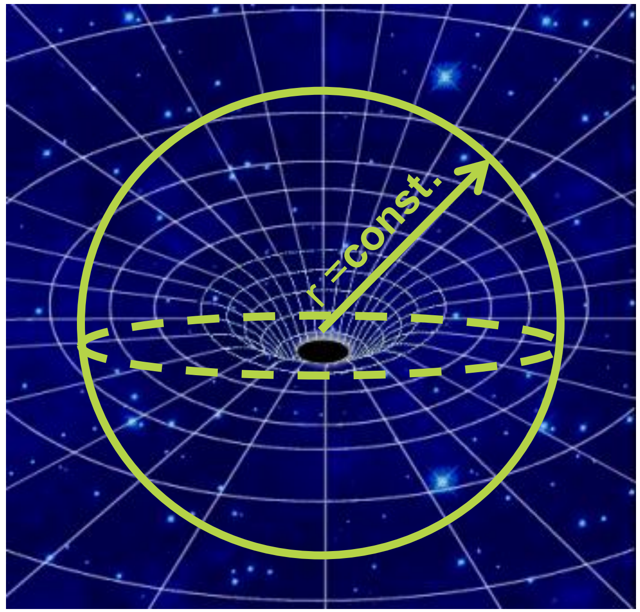
Helmholtz free energy – canonical ensemble

$$F[T,\ J] = M - T S\,,$$



Quasilocal thermodynamics

Due to the equivalence principle, there is no local definition of the energy in gravitational theories



Basic idea of the quasilocal energy: enclose a region of spacetime with some surface and compute the energy with respect to that surface – in fact all thermodinamical quantities can be computed in this way

Brown-York gr-qc/9209012

For asymptotical flat spacetime, it is possible to extend the quasilocal surface to spatial infinity provided one incorporates appropriate boundary (counterterms) in the action to remove divergences from the integration over the infinite volume of spacetime.

$$I = I_H[g] + I_B[g] + I_{ct}[h]$$

$$I = \frac{1}{16\pi G_5}\int_M R\sqrt{-g}\, d^5x + \frac{\epsilon}{8\pi G_5}\int_{\partial M}(K - c\sqrt{\mathcal{R}})\sqrt{-h}\, d^4x$$

$c = \sqrt{2}, \sqrt{3/2}$

$S^2 \times R \times R$ or $S^3 \times R$

Mann and Marolf

Compute directly the Gibbs-Duhem relation

$$\mathcal{G}[T, \Omega] = I/\beta$$

by integrating the action supported with counterterms.

Thermal stability

In analogy with the definitions for thermal expansion in the liquid-gas system, the specific heat at a constant angular velocity, the isothermal compressibility, and the coefficient of thermal expansion can be defined

$$C_\Omega = T\left(\frac{\partial S}{\partial T}\right)_\Omega = -T\left(\frac{\partial^2 \mathcal{G}}{\partial T^2}\right)_\Omega\,, \qquad \epsilon_T = \left(\frac{\partial J}{\partial \Omega}\right)_T\,, \qquad \alpha = \left(\frac{\partial J}{\partial T}\right)_\Omega\,.$$

The conditions for **thermal stability** in the grand-canonical ensemble

$$C_\Omega > 0\,, \qquad \epsilon_T > 0\,,$$

$$C_\Omega\,\epsilon_T - \alpha^2 T > 0$$

or

$$C_J > 0$$

The doubly spinning black hole and the singly spinning black ring are thermally unstable in the grand-canonical ensembles.

A second rotation could help to stabilize the solution

We investigated the stability of the doubly spinning black ring

Doubly spinning black ring

Investigate the thermodynamical stability for doubly spinning black rings

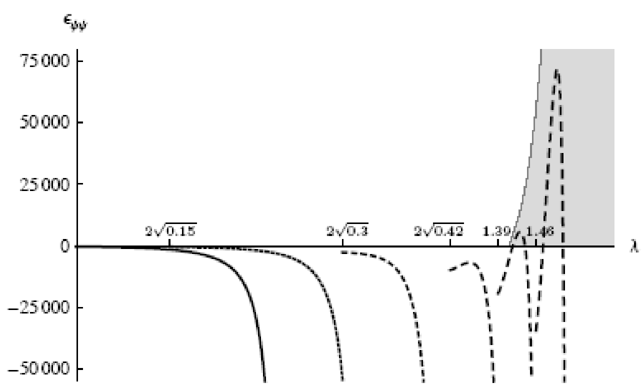
$$G[T,\Omega_\phi,\Omega_\psi] = \frac{\pi k^2}{G}\frac{\lambda}{(1+\nu-\lambda)}$$

The Hessian should be negatively defined

$$H(G) = (-1)\begin{pmatrix} C_\Omega T^{-1} & \alpha^a \\ \alpha^a & \epsilon^{ab} \end{pmatrix}$$

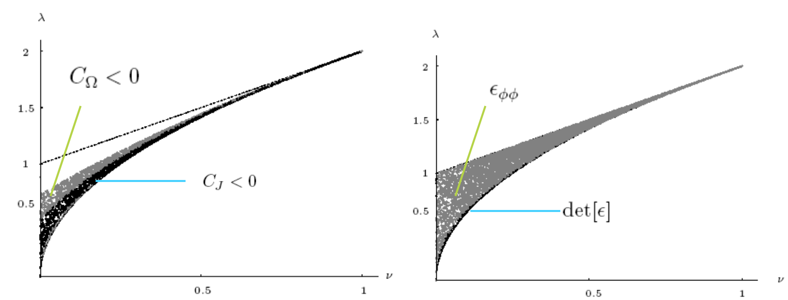
where

$$\alpha^a = \left(\frac{\partial J_i}{\partial T}\right)_\Omega \qquad \epsilon^{ab} = \left(\frac{\partial J_a}{\partial \Omega_b}\right)_T$$



Plot of the response function $\epsilon_{\phi\phi}$ as a function of λ.

As the angular momentum along S^2 is increased $\nu = 0.15, 0.3, 0.42, 0.48, 0.53$



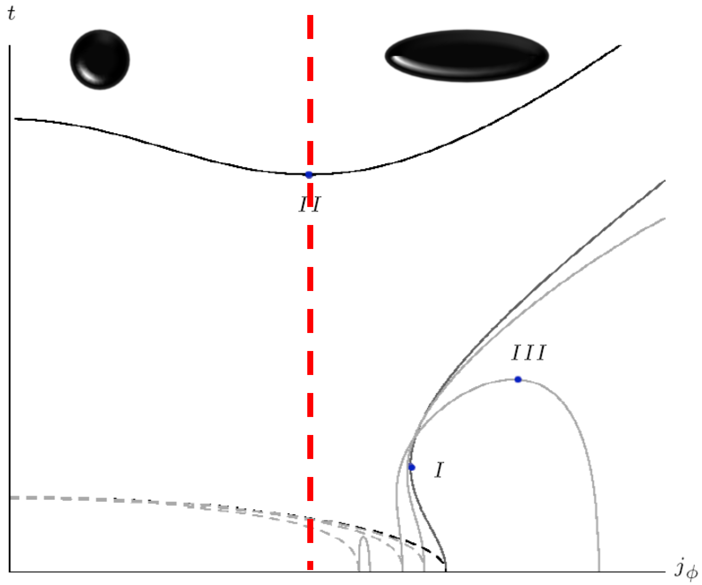
Scatter plots in parameter phase space (ν,λ) for the doubly spinning black ring.

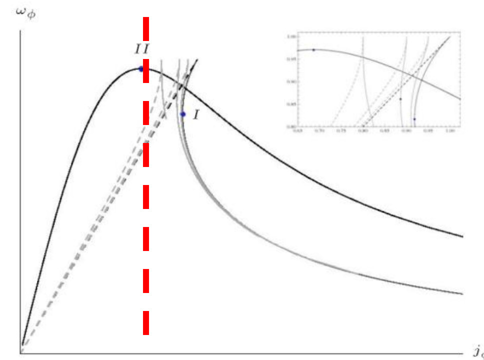
$$0 < \nu < 1\,, \qquad 2\sqrt{\nu} < \lambda < 1+\nu\,,$$

The doubly spinning black ring is local thermally unstable.

Instabilities from Thermodynamics

Critical points





$$\left(\frac{\partial^2 S}{\partial J^2}\right)_M = -\frac{1}{T}\left(\frac{\partial \Omega}{\partial J}\right)_M + \frac{\Omega}{T^2}\left(\frac{\partial T}{\partial J}\right)_M \quad \text{and} \quad \left(\frac{\partial^2 G}{\partial \Omega^2}\right)_T = -\left(\frac{\partial J}{\partial \Omega}\right)_T .$$

Black holes with one spin

$j_m \longrightarrow \frac{a^2}{r_h^2} = \frac{D-3}{D-5}$ where $\left(\frac{\partial^2 S}{\partial J^2}\right)_M = 0$

More general black holes with N spins ultra-spin iff

$$0 \leq a_1, a_2, ..., a_k << a_{k+1}, ..., a_N \to \infty .$$

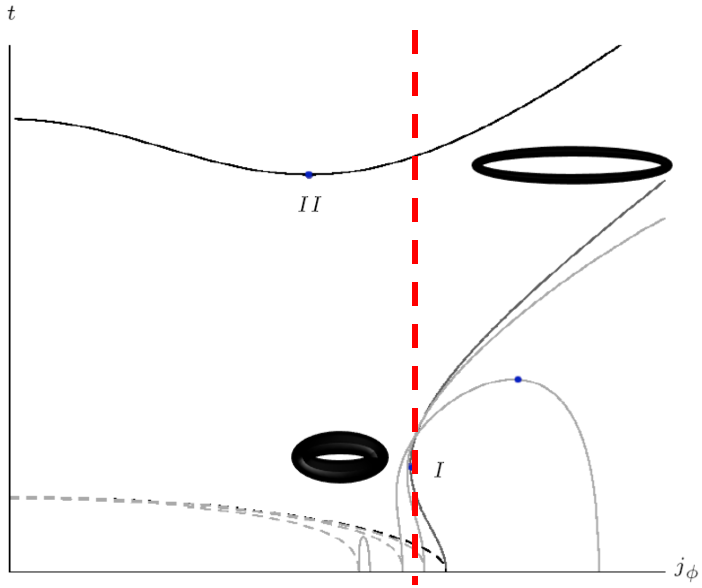
$$J_{k+1} = ... = J_N = J,$$

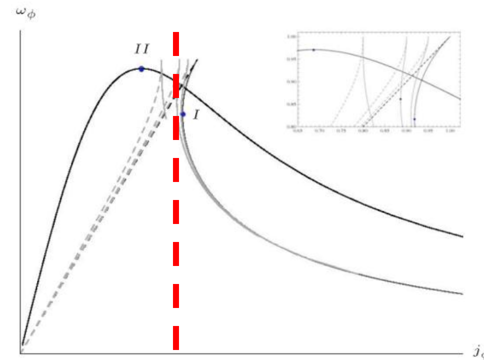
$$\frac{a^2}{r_h^2} = \frac{D-3}{2(k_o-1)} \text{ odd D}, \qquad \frac{a^2}{r_h^2} = \frac{D-3}{(2k_e-1)} \text{ even D}.$$

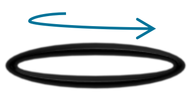
We checked that at least one of the eigenvalues of the Hess[G] is zero there.

Indicate where the transition to the black membrane phase.

Turning points

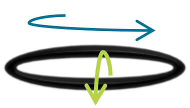






λ=0.5 At the cusp in s vs j

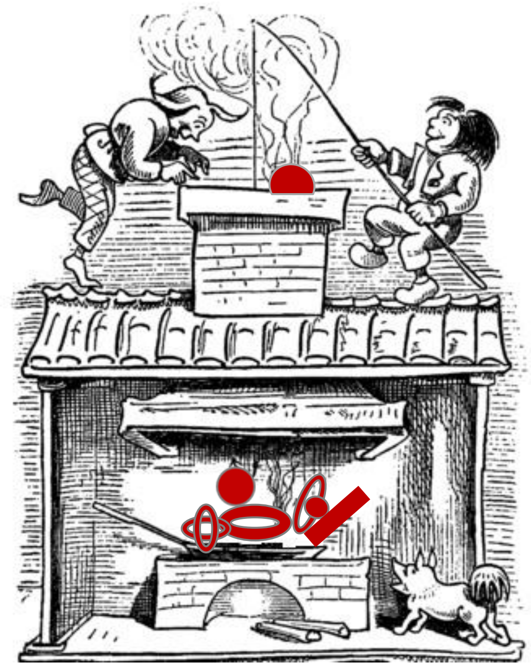
We checked that at least one of the eigenvalues of the Hess[G] is zero there.



λ [v] At the cusp in s vs j

Indicate where the transition to the black membrane phase.

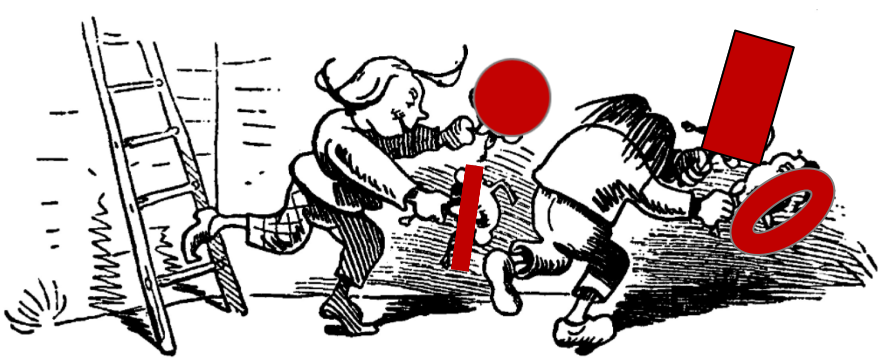
Summary and outlook



We showed that doubly spinning black rings are thermally unstable

Found the thresholds of the transition to the black membrane phase of black holes and black rings with at least two spins.

It will be interesting to investigate numerically whether these correspond to the zero-mode perturbations.



Danke.