Exotic objects from DFT

based on the work with A. Kleinschmidt and I. Bakhmatov
[1607.05450]+JHEP

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T-duality

- Mass spectrum of a closed string on a space $M \times \mathbb{S}^1$ is invariant under

$$R \leftrightarrow \frac{1}{R} \quad \text{T-duality}$$

- On the level of supergravity fields this translates into Buscher rules
- For a torus $\mathbb{T}^d$ T-duality group is $O(d, d)$ and acts as

$$\begin{aligned} g + B = E \Longrightarrow \; E' = \frac{aE + b}{cE + d}, \\ 2d \times 2d \text{ matrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in O(d, d) \end{aligned} \tag{1}$$

- Extended objects of string theory transform as

$$\begin{aligned} D_p \Longrightarrow D_{p-1} \oplus D_{p+1}, \\ NS5 \Longrightarrow KK \Longrightarrow \dots \end{aligned} \tag{2}$$

Flux compactifications

- Generalization of Buscher rules

$$
\begin{aligned}
&F_{m_1\dots m_p x} \overset{T_x}{\longleftrightarrow} F_{m_1\dots m_p} \\
&H_{xyz} \overset{T_x}{\longleftrightarrow} f^x{}_{yz} \overset{T_y}{\longleftrightarrow} Q^{xy}{}_z \overset{T_z}{\longleftrightarrow} R^{xyz}.
\end{aligned}
\tag{3}
$$

[Schelton, Taylor, Wecht]

- Non-geometric fluxes (gaugings) do not correspond to any geometric data of the compact manifold;
- Gauged supergravity: gaugings (topological fluxes) are necessary to stabilize moduli;
- Sources of non-geometric fluxes – non-geometric branes

Supergravity on a twisted torus: a $\mathbb{T}^2$ fibration over $\mathbb{S}^1$

- Field strength $H_{abc} \neq 0$ (NS5-brane)

$$ds^2 = dx^2 + dy^2 + dz^2, \quad B = Nz dx \wedge dy,$$

$$z \to z + 1, \quad B \to B + d\lambda$$

T_x

- Torsion $f^a{}_{bc} \neq 0$ (Kaluza-Klein monopole)

$$ds^2 = (dx + Nz dy)^2 + dy^2 + dz^2, \quad B = 0,$$

$$z \to z + 1, \quad x \to x + Ny,$$

T_y

- Non-geometric flux $Q^{ab}{}_c \neq 0$ (5^2_2-brane)

$$ds^2 = \frac{1}{1 + N^2 z^2}(dx^2 + dy^2 + dz^2), \quad B = \frac{Nz}{1 + N^2 z^2} dx \wedge dy.$$

$$z \to z + 1, \quad \text{glued by T-duality}$$

[Shelton, Taylor, Wecht]

T-duality orbit of NS branes

Asymtpotically well defined solutions

- NS5-brane, $H(R) = 1 + \frac{h}{R^2}, \quad R^2 = \delta_{ij}x^i x^j + (x^4)^2, \quad \{x^1, x^2, x^3\}$

$$\begin{aligned} ds^2 &= H(R)\Big((dx^4)^2 + \delta_{ij}dx^i dx^j\Big) + ds^2_{(1,5)}, \\ B &= A_i dx^i \wedge dx^4, \\ 2\partial_{[i}A_{j]} &= \varepsilon_{ijk}\partial_k H(R) \end{aligned} \tag{4}$$

- KK monopole, $H(r) = 1 + \frac{h}{r}, \quad r^2 = \delta_{ij}x^i x^j, \quad x^4$ is compact

$$\begin{aligned} ds^2 &= H(r)^{-1}\Big((dx^4)^2 + A_i dx^i\Big)^2 + H(r)\delta_{ij}dx^i dx^j + ds^2_{(1,5)}, \\ B &= 0. \end{aligned} \tag{5}$$

[deBoer, Shigemori]

Non-geometric backgrounds

T-duality along compact x^3 ends up with a not well defined background

- 5^2_2-brane, $H(\rho) = 1 + h \log\frac{\mu}{\rho}$, $\rho^2 = (x^1)^2 + (x^2)^2$

$$\begin{aligned} ds^2 &= \frac{H}{H^2 + h^2\theta^2}\Big((dx^4)^2 + (dx^3)^2\Big) + H\Big((dx^1)^2 + (dx^2)^2\Big) + ds^2_{(1,5)}, \\ B &= \frac{h\theta}{H^2 + h^2\theta^2} dx^4 \wedge dx^3 \end{aligned} \tag{6}$$

- Non-trivial monodromy around the brane $\theta \to \theta + 2\pi \implies$
- Non-commutativity of string coordinates
- This is supposed to have Q-flux $Q_\theta{}^{43} = h$

Monodromy of the 5^2_2-brane

- Define the object

$$\mathcal{H} = \begin{bmatrix} g + Bg^{-1}B & Bg^{-1} \\ g^{-1}B & g^{-1} \end{bmatrix} \tag{7}$$

- Monodromy around the brane

$$\mathcal{H}(\theta + 2\pi) = \mathcal{O}^{\text{tr}}\mathcal{H}(\theta)\mathcal{O}, \quad \mathcal{O} = \begin{bmatrix} 1_2 & 0 \\ \beta(\theta) & 1_2 \end{bmatrix} \tag{8}$$

- Patched by a β-transformation

$$\beta = h\theta\partial_4 \wedge \partial_3 \tag{9}$$

The non-trivial monodromy around the brane implies that the space is non-commutative.

Beta-supergravity

- Field redefinition of beta-supergravity

$$(g+B)^{-1} = \hat{g}^{\,-1} + \beta \tag{10}$$

- The background of 5^2_2-brane becomes

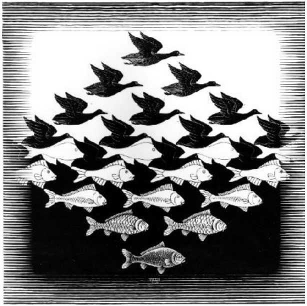
$$\begin{aligned} d\hat{s}^2 &= H(r)\Big((dx^1)^2 + (dx^2)^2 + (dx^4)^2 + (dx^3)^2\Big) + ds^{\,2}_{(1,5)}, \\ \beta &= h\theta \partial_4 \wedge \partial_3, \end{aligned} \tag{11}$$

- The flux is defined by $Q_m{}^{nk} = \partial_m \beta^{nk}$
- Magnetic coupling with 8-form with values in $\wedge^2$ TM

$$\beta^{mn} \Longleftrightarrow B_{(8)}{}^{mn} \tag{12}$$

[Andriot, Betz, Chatzistavrakidis, Hohm, Lüst, Zagermann, ...]

Double Field Theory



The idea

- Consider objects transforming covariantly under duality symmetries
- Construct a geometry consistent with duality symmetries;
- Formulate a field theory where the duality symmetries appear geometrically;

Solutions of DFT unify known supergravity solutions

- DFT wave solution = an F1 string or pp-wave,
- DFT monopole solution = NS5 brane, KK monopole, ...

Doubled geometry

- Momentum and winding modes of a string unify into a doubled momentum;

$$\mathcal{P}^M = \begin{bmatrix} p^m \\ w_n \end{bmatrix} \Longrightarrow \mathbb{X}^M = \begin{bmatrix} x^m \\ \tilde{x}_m \end{bmatrix} \text{ doubled coordinate} \tag{13}$$

- Mass spectrum is invariant under $O(d,d)$;

$$\begin{aligned} M^2 &= \mathcal{P}^M \mathcal{H}_{MN} \mathcal{P}^N, \\ &\text{generalized metric:} \\ \mathcal{H}_{MN} &= \begin{bmatrix} g - Bg^{-1}B & Bg^{-1} \\ g^{-1}B & g^{-1} \end{bmatrix}, \quad \mathcal{H} \to \mathcal{O}^T \mathcal{H} \mathcal{O}, \quad \mathcal{O} \in O(d,d) \end{aligned} \tag{14}$$

- Invariant dilaton

$$d = \phi + \frac{1}{4} \log g \tag{15}$$

Doubled geometry

- diffeos $\oplus$ gauge transformations $\Longrightarrow$ transformations of doubled space

$$\begin{aligned} L_\Sigma T^M &= \text{Shift}[T] + GL(n)\text{-rotation}[T] && \text{Riemannian geometry} \\ \mathcal{L}_\Sigma T^M &= \text{Shift}[T] + O(d,d)\text{-rotation}[T]. && \text{Doubled geometry} \end{aligned} \tag{16}$$

- Consistency of the algebra $\Longrightarrow$ section condition:

$$\frac{\partial}{\partial x^m} T_1 \cdot \frac{\partial}{\partial \tilde{x}_m} T_2 = 0, \quad \forall\, T_{1,2} - \text{generalized tensors} \tag{17}$$

- Effective potential

$$\begin{aligned} V = e^{-2d}\Big[&\frac{1}{8}\mathcal{H}^{MN}(\partial_M \mathcal{H}_{KL})(\partial_N \mathcal{H}^{KL}) - \frac{1}{2}\mathcal{H}^{MN}(\partial_N \mathcal{H}^{KL})(\partial_L \mathcal{H}_{MK}) - \\ &- 2\partial_M d \partial_N \mathcal{H}^{MN} + 4\mathcal{H}^{MN}\partial_M d \partial_N d\Big] \end{aligned} \tag{18}$$

[Berman, Cederwall, Coimbra, Godazgar^2, Graña, Hohm, Hull, ETM, Nicolai, Perry, Samtleben, Thompson, Waldram, Zwiebach ...]

Section condition

- The section condition halves the number of coordinates;
- Different ways of doing that are related by T-duality

$$T_x : \quad x \longleftrightarrow \tilde{x} \tag{19}$$

- The action drops to that of supergravity

$$V = \sqrt{g}\, e^{2\phi}\left(R - \frac{1}{12}H_{mnp}H^{mnp} + 4g^{mn}\partial_m\phi\partial_n\phi\right) \tag{20}$$

- The whole construction reproduces Hitchin's generalized geometry

$$\begin{aligned}
&\text{Doubled tangent space:} \quad \Xi_p = X_p \oplus \xi_p \in T_pM \oplus T_p^*M \\
&\delta_{X\oplus\xi}G = L_X G, \quad \delta_{X\oplus\xi}B = L_X B + d\xi \\
&\mathcal{L}_{\Xi_1}\Xi_2 - \mathcal{L}_{\Xi_2}\Xi_1 \longrightarrow [\Xi_1, \Xi_2]_C
\end{aligned} \tag{21}$$

Generalised flux

Generalised flux is defined as a generalised torsion

$$[e_a, e_b] = f^a{}_{bc} e_a \quad \Longrightarrow \quad [E_A, E_B]_C = F^C{}_{AB} E_C.$$

$$[V_1, V_2]_C = \frac{1}{2}\Big(\mathcal{L}_{V_1} V_2 - \mathcal{L}_{V_2} V_1\Big). \tag{22}$$

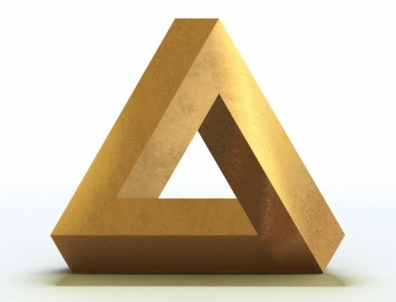
Its components are naturally identified with fluxes

$$f^a{}_{bc} = \mathcal{F}^a{}_{bc}, \quad H_{abc} = \mathcal{F}_{abc}, \quad Q^{ab}{}_c = \mathcal{F}^{ab}{}_c, \quad R^{abc} = \mathcal{F}^{abc}. \tag{23}$$

DFT EOM's and consistency conditions imply generalised Bianchi identities

$$\partial \mathcal{F} + \mathcal{F} \otimes \mathcal{F} = 0 \text{ (or } \delta\text{-function)} \tag{24}$$

Non-geometric branes in DFT



DFT monopole

Kaluza-Klein understanding of the theory

$$\begin{aligned} ds^2_{DFT} &= \mathcal{H}_{MN} d\mathbb{X}^M d\mathbb{X}^N \\ &= (g_{\mu\nu} - B_\mu{}^\rho B_{\rho\nu}) dx^\mu dx^\nu + 2B_\mu{}^\nu dx^\mu d\tilde{x}_\nu + g^{\mu\nu} d\tilde{x}_\mu d\tilde{x}_\nu . \end{aligned} \tag{25}$$

The Taub-NUT-like solution in coordinates $(z, y^i, x^a, \tilde{z}, \tilde{x}_i, \tilde{x}_a)$

$$\begin{aligned} ds^2_{DFT} &= H(1 + H^{-2}A^2) dz^2 + H^{-1} d\tilde{z}^2 + 2H^{-1} A_i (dy^i d\tilde{z} - \delta^{ij} d\tilde{y}_j dz) \\ &\quad + H(\delta_{ij} + H^{-2} A_i A_j) dy^i dy^j + H^{-1} \delta^{ij} d\tilde{y}_i d\tilde{y}_j \\ &\quad + \eta_{ab} dx^a dx^b + \eta^{ab} d\tilde{x}_a d\tilde{x}_b , \\ e^{-2d} &= He^{-2\phi_0} . \end{aligned} \tag{26}$$

With harmonic function

$$H(y) = 1 + \frac{h}{\sqrt{\delta_{ij} y^i y^j}}, \quad 2\partial_{[i} A_{j]} = \varepsilon_{ijk} \partial_k H, \tag{27}$$

[Berman, Rudolph]

Physical subspace

- The doubled coordinates $\mathbb{X}^M = (x^z, x^i, x^a, \tilde{x}_z, \tilde{x}_i, \tilde{x}_a)$ are identified with the parameters $z, y^i, \tilde{z}, \tilde{y}_i$ with a high level of ambiguity
- The choice matters

$$
\begin{aligned}
(x^z, x^1, x^2, x^3) &= (z, y^1, y^2, y^3), && \text{NS5-brane (H-monopole)},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, y^1, y^2, y^3), && \text{KK-monopole},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, y^1, y^2, \tilde{y}^3), && \text{Q-monopole},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}^1, \tilde{y}^2, y^3), && \text{R-monopole},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}^1, \tilde{y}^2, \tilde{y}^1), && \text{R'-monopole},
\end{aligned}
\tag{28}
$$

- The harmonic function is always

$$
H(y^1, y^2, y^3) = 1 + \frac{h}{\sqrt{\delta_{ij} y^i y^j}} \tag{29}
$$

- Non-geometric backgrounds depend on dual coordinates

T-duality orbit of NS branes

- NS5-brane, $H(R) = 1 + \frac{h}{R^2}$, $R^2 = \delta_{ij}x^i x^j + (x^4)^2$, $\{x^1, x^2, x^3\}$

$$
\begin{aligned}
& ds^2 = H(R)\Big((dx^4)^2 + \delta_{ij}dx^i dx^j\Big) + ds^2_{(1,5)}, \quad B = A_i dx^i \wedge dz, \\
& 2\partial_{[i}A_{j]} = \varepsilon_{ijk}\partial_k H(R)
\end{aligned} \tag{30}
$$

- KK monopole, $H(r) = 1 + \frac{h}{r}$, $r^2 = \delta_{ij}x^i x^j$

$$
ds^2 = H(r)^{-1}\Big((dx^4)^2 + A_i dx^i\Big)^2 + H(r)\delta_{ij}dx^i dx^j + ds^2_{(1,5)}, \tag{31}
$$

- 5^2_2-brane, $H(\rho) = 1 + h \log\frac{\mu}{\rho}$, $\rho^2 = (x^1)^2 + (x^2)^2$

$$
\begin{aligned}
& ds^2 = \frac{H}{H^2 + h^2\theta^2}\Big((dx^4)^2 + (dx^3)^2\Big) + H\Big((dx^1)^2 + (dx^2)^2\Big) + ds^2_{(1,5)}, \\
& B = \frac{h\theta}{H^2 + h^2\theta^2} dx^4 \wedge dx^3
\end{aligned} \tag{32}
$$

[deBoer, Shigemori]

H- and KK-monopoles

- H-monopole. Physical: (z, y^1, y^2, y^3)

$$
\begin{aligned}
ds^2 &= \eta_{rs} dx^r dx^s + H(dz^2 + \delta_{ij} dy^i dy^j), \\
B &= A_i dy^i \wedge dz, \\
H &= 1 + \frac{h}{\sqrt{(y^1)^2 + (y^2)^2 + (y^3)^2}}
\end{aligned}
\tag{33}
$$

- KK-monopole. Physical: $(\tilde{z}, y^1, y^2, y^3)$

$$
\begin{aligned}
ds^2 &= \eta_{rs} dx^r dx^s + H^{-1}(d\tilde{z} + A_i dy^i)^2 + H\delta_{ij} dy^i dy^j, \\
B &= 0.
\end{aligned}
\tag{34}
$$

Q-monopole

- Physical: $(\tilde{z}, y^1, y^2, \tilde{y}_3)$

$$
\begin{aligned}
ds^2 &= \eta_{rs} dx^r dx^s + \frac{H}{H^2 + A_3^2}\Big((d\tilde{z} + A_\alpha dy^\alpha)^2 + d\tilde{y}_3^2\Big) + H\delta_{\alpha\beta} dy^\alpha dy^\beta, \\
B &= \frac{A_3}{H^2 + A_3^2}(d\tilde{z} + A_\alpha dy^\alpha) \wedge d\tilde{y}_3, \\
H &= 1 + \frac{h}{\sqrt{\delta_{\alpha\beta} y^\alpha y^\beta + (y^3)^2}}.
\end{aligned}
\tag{35}
$$

- Conditions:

$$
\begin{aligned}
2\partial_{[i} A_{j]} &= \varepsilon_{ijk} \partial_k H, \\
\partial_i A_i &= 0, \\
\triangle H &= 0
\end{aligned}
\tag{36}
$$

Q-monopole

- Physical: $(\tilde{z}, y^1, y^2, \tilde{y}_3) \Longrightarrow$ axial coordinates:

$$\begin{aligned} y^1 &= \rho \cos\theta, \\ y^2 &= \rho \sin\theta, \\ y^3 &= y^3 \text{ (the dual)}. \end{aligned} \tag{37}$$

- The Q-monopole background:

$$\begin{aligned} ds^2 &= H^{-1}\left[\left(d\tilde{z} + A_\theta d\theta\right)^2 + d\tilde{y}_3{}^2\right] + H\left(d\rho^2 + \rho^2 d\theta^2\right), \\ H &= 1 + \frac{h}{\sqrt{\rho^2 + (y^3)^2}} \\ A_\theta &= h\left(1 - \frac{y^3}{\sqrt{\rho^2 + (y^3)^2}}\right). \end{aligned} \tag{38}$$

- No monodromy $\Longrightarrow$ no non-commutativity

R-monopole

- Physical: $(\tilde z, y^1, \tilde y_2, \tilde y_3) \Longrightarrow$ axial coordinates:

$$\begin{aligned} & y^1 \\ & \tilde y_2 = \rho\cos\theta, \quad \tilde x_2 = y^2 = \tilde\rho\cos\tilde\theta \\ & \tilde y_3 = \rho\sin\theta, \quad \tilde x_3 = y^3 = \tilde\rho\sin\tilde\theta \end{aligned} \tag{39}$$

- The R-monopole background (β − frame):

$$\begin{aligned} & ds^2 = \eta_{rs}dx^r dx^s + H^{-1}\left(d\tilde z^2 + d\rho^2 + \rho^2 d\theta^2\right) + H(dy^1)^2, \\ & \beta^{\theta z} = A_\theta, \\ & A_\theta = h\left(1 - \frac{y^1}{\sqrt{\tilde\rho^2 + (y^1)^2}}\right), \quad H = 1 + \frac{h}{\sqrt{\tilde\rho^2 + (y^1)^2}}. \end{aligned} \tag{40}$$

- No monodromy $\Longrightarrow$ no non-associativity

Generalized fluxes

Generalized fluxes

$$[E_A, E_B] = F_{AB}{}^C E_C, \qquad \mathcal{F}_A = \partial_M E_A^M + 2E_A^M \partial_M d \tag{41}$$

- H-monopole: $$\mathcal{H}_{\bar z\bar a\bar b} = 2e^z{}_{\bar z} e^k{}_{\bar a} e^l{}_{\bar b} \partial_{[k} A_{l]}, \quad \mathcal{F}^{\bar a}{}_{\bar b\bar c} \Longleftrightarrow \mathcal{F}_{\bar a} \tag{42}$$
- KK-monopole: $$\mathcal{F}^{\bar a}{}_{\bar b\bar c} = 2e^{\bar a}{}_z e^k{}_{\bar b} e^l{}_{\bar c} \partial_{[k} A_{l]} - 2\delta^{\bar a}{}_{[\bar b} \mathcal{F}_{\bar c]}, \quad \mathcal{F}_{\bar a} \tag{43}$$
- Q-monopole:

$$Q_{\bar\alpha}{}^{\bar 3\bar z} = \varepsilon_{\bar\alpha\bar\beta} H^{-1} \partial_{\bar\beta} H, \quad Q_{\bar\alpha}{}^{\bar\beta\bar 3} \Longleftrightarrow Q_{\bar z}{}^{\bar z\bar 3} \Longleftrightarrow \mathcal{F}^{\bar 3}, \quad \mathcal{F}_{\bar m\bar n}{}^{\bar p} \Longleftarrow \mathcal{F}_{\bar k}. \tag{44}$$

- R-monopole:

$$R^{\bar z\bar\alpha\bar\beta} = \varepsilon^{\bar\alpha\bar\beta} H^{-\frac{3}{2}} \partial_1 H, \quad \mathcal{F}_{\bar m\bar n}{}^{\bar p} \Longleftrightarrow Q_{\bar m\bar n}{}^{\bar p} \Longleftrightarrow \mathcal{F}_{\bar k} \tag{45}$$

Sources and Bianchi identities

The RHS of Bianchi identities tells something about source of a solution

$$\begin{aligned}
H: &\quad \partial_{[a}H_{bcd]} + \ldots = \frac{\alpha}{4H^2}\varepsilon^{1234}\varepsilon_{abcd}\delta\big((x^1)^2+(x^2)^2+(x^3)^2\big),\\
KK: &\quad \partial_{[b}f^{a}{}_{cd]} + \ldots = \frac{\alpha}{H^2}\varepsilon^{a123}\varepsilon_{bcd4}\delta\big((x^1)^2+(x^2)^2+(x^3)^2\big),\\
Q: &\quad \partial_{[c}Q_{d]}{}^{ab} + \ldots = -\frac{\alpha}{H^2}\varepsilon^{ab12}\varepsilon_{cd34}\delta\big((x^1)^2+(x^2)^2+(\tilde{x}^3)^2\big),\\
R: &\quad \partial_{d}R^{abc} + \ldots = -\frac{\alpha}{H^2}\varepsilon^{abc1}\varepsilon_{d234}\delta\big((x^1)^2+(\tilde{x}^2)^2+(\tilde{x}^3)^2\big).
\end{aligned} \tag{46}$$

Brane support from BI

$$\begin{aligned}
\text{Action}: &\quad S = \int H_7 \wedge *H_7 + \int B_6 \wedge \delta_4;\\
\text{EOM}: &\quad d * H_7 = \delta_4
\end{aligned} \tag{47}$$

Non-perturbative effects

T-duality: KK-monopole $\Longleftrightarrow$ H-monopole.

- Adding worldsheet instanton corrections leads to localized NS5-brane

$$\begin{aligned} H(x^i, z) &= 1 + \frac{h}{2r}\left(1 + \sum_{k=1}^{\infty} e^{-kr+ikz} + \sum_{k=1}^{\infty} e^{-kr-ikz}\right) \\ &= 1 + \frac{h}{2r}\frac{\sinh r}{\cosh r - \cos z} \end{aligned} \tag{48}$$

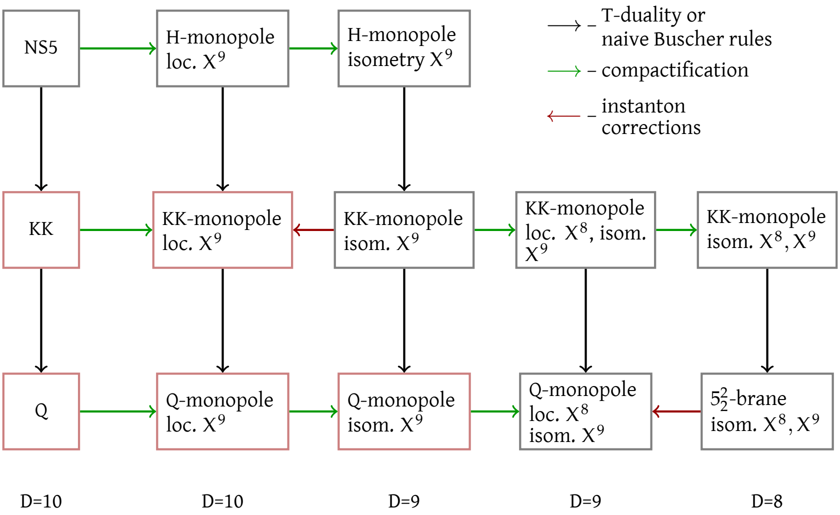
- Smearing procedure of the harmonic function of the full NS5-brane

$$H(x^i, z) = 1 + \sum_{k=-\infty}^{\infty} \frac{h}{r^2 + (z + 2\pi k)^2} \tag{49}$$

- For KK-monopole and 5^2_2-brane this introduces dependence on winding coordinates

[Tong, Jensen, Harvey, Kimura, Sasaki]

Integration into the general picture



[Jensen, Harvey, Gregory, Moore, Kimura, Sasaki]

Magnetic charge

- Charges in gauge theory

$$4\pi q = \int \partial_i E^i dV = \int_\Sigma *F, \quad 4\pi\mu = \int \partial_i B^i dV = \int_\Sigma F, \tag{50}$$

- Integration surface $\Sigma = \mathbb{S}^2 \times \mathbb{S}^1$

Σ	x^1	x^2	x^3	x^z	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_z$
H	×	×	×	•				
KK	×	×	×					•
Q	×	×					×	•
R	×					×	×	•

- Duality-invariant magnetic charge

$$4\pi\mu_{DFT} = \int_\Sigma \mathcal{F}_{MNK} d\mathbb{X}^M \wedge d\mathbb{X}^N \wedge d\mathbb{X}^K = 4\pi Q_{NS5}, \tag{51}$$

Future directions

- Check to what extent these backgrounds are solutions of the Type II equations of motion and BPS equation
- Construct effective action for these objects
- Compactifications with such sets of fluxes, tadpole cancellation conditions.
- Properly define electric charge
- Non-geometric branes of xFT, classification
- Genuine non-geometric branes

Thank you!