Non-Supersymmetric Magic Theories and Ehlers Truncations

based on 1701.03031 with Alessio Marrani, Gianfranco Pradisi and Fabio Riccioni

Luca Romano

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Leibniz Universitaet Hannover

1. Magic Theories

2. Ehlers Truncations

3. Conclusions ans Outlook

Freudenthal Magic Square

$\mathbb{A} \setminus \mathbb{B}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$\mathfrak{so}(3)$	$\mathfrak{su}(3)$	$\mathfrak{sp}(3)$	$\mathfrak{f}_4$
$\mathbb{C}$	$\mathfrak{su}(3)$	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$\mathfrak{su}(6)$	$\mathfrak{e}_6$
$\mathbb{H}$	$\mathfrak{sp}(3)$	$\mathfrak{su}(6)$	$\mathfrak{so}(12)$	$\mathfrak{e}_7$
$\mathbb{O}$	$\mathfrak{f}_4$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_8$

Tit's approach

$$\mathfrak{g} = \left[\mathfrak{der}(\mathbb{A}) \oplus \mathfrak{der}\left(\mathfrak{J}_3^{\mathbb{B}}\right)\right] \oplus \left[\mathbb{A}_0 \times (\mathfrak{J}_3^{\mathbb{B}})_0\right]$$

- $\mathfrak{der}(\mathbb{A})$ — *automorphisms of the division algebra* $\mathbb{A}$
- $\mathfrak{der}(\mathfrak{J}_3^{\mathbb{B}})$ — *automorphisms of the Jordan algebra* $\mathfrak{J}_3^{\mathbb{B}}$
- $\mathbb{A}_0$ — *traceless elements in* $\mathbb{A}$
- $(\mathfrak{J}_3^{\mathbb{B}})_0$ — *traceless elements in* $\mathfrak{J}_3^{\mathbb{B}}$

Jordan Algebras

Jordan Algebra

A Jordan algebra $\mathfrak{J}$ over a field $\mathbb{F}$ (not of characteristic 2) is an algebra over $\mathbb{F}$ equipped with a product operation $\circ : \mathfrak{J} \times \mathfrak{J} \to \mathfrak{J}$ satisfying

- $A \circ B = B \circ A$ (*commutative*)
- $A^2 \circ (A \circ B) = A \circ (A^2 \circ B)$ (*not associative*)

$\forall A, B \in \mathfrak{J}$.$\circ$ is called Jordan product.

Cubic norm

Let V be a vector space and $\mathbb{F}$ a field, a cubic norm is a map $N : V \to \mathbb{F}$ satisfying

$$N(\alpha A) = \alpha^3 N(A) \qquad \forall A \in V, \alpha \in \mathbb{F}$$

such that its linearization

$$N(A,B,C) \equiv \frac{1}{6}\left[N(A+B+C) - N(A+B) - N(A+C) - N(B+C) + N(A) + N(B) + N(C)\right]$$

is trilinear.

Jordan Algebras

Automorphism Group

The automorphism group of a Jordan algebra $\mathfrak{J}$ is the set of $\mathbb{R}$-linear transformations τ preserving the Jordan product

$$\mathrm{Aut}(\mathfrak{J}) \equiv \{\tau \in \mathrm{Iso}_{\mathbb{R}}(\mathfrak{J}) \mid \tau(A \circ B) = \tau A \circ \tau B\}$$

the corresponding Lie algebra is given by

$$\mathfrak{Aut}(\mathfrak{J}) \sim \mathfrak{der}(\mathfrak{J}) = \{\delta \in \mathrm{Hom}_{\mathbb{R}}(\mathfrak{J}) \mid \delta(\mathrm{A} \circ \mathrm{B}) = \delta \mathrm{A} \circ \mathrm{B} + \mathrm{A} \circ \delta \mathrm{B}\}$$

Reduced Structure Group

The reduced structure group of a Jordan algebra $\mathfrak{J}$ is the set of $\mathbb{R}$-linear transformations τ preserving the cubic norm

$$\mathrm{Str}_0(\mathfrak{J}) \equiv \{\tau \in \mathrm{Iso}_{\mathbb{R}}(\mathfrak{J}) \mid N(\tau \mathrm{A}) = N(\mathrm{A})\}$$

the corresponding Lie algebra is given by

$$\mathfrak{Str}_0(\mathfrak{J}) = \{\phi \in \mathrm{Hom}_{\mathbb{R}}(\mathfrak{J}) \mid N(\phi \mathrm{A}, \mathrm{A}, \mathrm{A}) = 0\}$$

Cubic Jordan Algebra

Cubic Jordan Algebra $\mathfrak{J}_3^{\mathbb{B}}$

$\mathfrak{J}_3^{\mathbb{B}}$ is the Jordan algebra of 3×3 Hermitian matrices over $\mathbb{B}$.

$$A = \begin{pmatrix} a & \alpha & \beta \\ \alpha^* & b & \gamma \\ \beta^* & \gamma^* & c \end{pmatrix} \qquad a, b, c \in \mathbb{R} \qquad \alpha, \beta, \gamma \in \mathbb{B}$$

Jordan product $\quad A \circ B = \frac{1}{2}(AB + BA)$

Cubic Norm $\quad N(A) = \det(A) \qquad (\textit{for } \mathbb{R}, \mathbb{C})$

Split Magic Square

$\mathbb{A} \setminus \mathbb{B}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$	
$\mathbb{R}$	$\mathfrak{so}(3)$	$\mathfrak{su}(3)$	$\mathfrak{usp}(6)$	$\mathfrak{f}_4$	
$\mathbb{C}_s$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{C})$	$\mathfrak{su}^*(6)$	$\mathfrak{e}_{6(-26)}$	*dim* 5
$\mathbb{H}_s$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{su}(3,3)$	$\mathfrak{so}^*(12)$	$\mathfrak{e}_{7(-25)}$	*dim* 4
$\mathbb{O}_s$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(2)}$	$\mathfrak{e}_{7(-5)}$	$\mathfrak{e}_{8(-24)}$	*dim* 3

$\mathcal{N} = 2$ MAxwell Einsten supergavity

The magic square contains the symmetry of the $\mathcal{N} = 2$ Maxwell Einstein theories in D=3, 4, 5.

$$D = 5 \qquad \mathcal{M} = \mathfrak{Str}_0(\mathfrak{J})/\mathfrak{Aut}(\mathfrak{J})$$

Exceptional supergravity theories and the magic square, *Günaydin M. , Sierra G. and Townsend, P.K.* Physics Letters B, Volume 133, Issue 1-2, p. 72-76.

Split Magic Square

$\mathbb{A} \setminus \mathbb{B}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$	
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$\mathbb{C}_s$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{C})$	$\mathfrak{su}^*(6)$	$\mathfrak{e}_{6(-26)}$	*dim* 5
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$\mathcal{N} = 2$ Maxwell Einsten supergavity

The magic square contains the symmetry of the $\mathcal{N} = 2$ Maxwell Einstein theories in D=3, 4, 5.

$$\ldots + C_{IJK}\ F_2^I \wedge F_2^J \wedge A_1^K + \ldots$$

Invariant tensor of the symmetry group

The Geometry of N=2 Maxwell-Einstein Supergravity and Jordan Algebras, *Günaydin M. , Sierra G. and Townsend, P.K.* Nucl. Phys. B Volume 242, 1984,p. 244-268

Split Magic Square and $\mathcal{N}=2$ Maxwell Einsten Theories

$\mathbb{A}\setminus\mathbb{B}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$\mathfrak{so}(3)$	$\mathfrak{su}(3)$	$\mathfrak{usp}(6)$	$\mathfrak{f}_4$
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Magic Square

$$\mathfrak{g}=\left[\mathfrak{der}(\mathbb{A})\oplus\mathfrak{der}\left(\mathfrak{J}_3^{\mathbb{B}}\right)\right]\oplus\left[\mathbb{A}_0\times(\mathfrak{J}_3^{\mathbb{B}})_0\right]$$

$\mathcal{N}=2$ Supergravity

$D=5$ $\quad \mathcal{M}=\mathfrak{Str}_0(\mathfrak{J})/\mathfrak{Aut}(\mathfrak{J})$

$D=4$ $\quad \mathcal{M}=\mathfrak{Mob}_0(\mathfrak{J})/\widetilde{\mathfrak{Str}}(\mathfrak{J})$

$D=3$ $\quad \ldots$

Jordan Algebra

$\mathfrak{J}_3^{\mathbb{B}}$

Exceptional supergravity theories and the magic square, *Günaydin M. , Sierra G. and Townsend, P.K.* Physics Letters B, Volume 133, Issue 1-2, p. 72-76.

Doubly Split Magic Square

Another version of the magic Square could be constructed taking both $\mathbb{A}, \mathbb{B}$ in their split form: the doubly split magic square

$\mathbb{A} \setminus \mathbb{B}$	$\mathbb{R}$	$\mathbb{C}_s$	$\mathbb{H}_s$	$\mathbb{O}_s$	
$\mathbb{R}$	$\mathfrak{so}(3)$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{f}_{4(4)}$	
$\mathbb{C}_s$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{R}) \times \mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(6,\mathbb{R})$	$\mathfrak{e}_{6(6)}$	*dim* 5
$\mathbb{H}_s$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{sl}(6,\mathbb{R})$	$\mathfrak{so}(6,6)$	$\mathfrak{e}_{7(7)}$	*dim* 4
$\mathbb{O}_s$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(6)}$	$\mathfrak{e}_{7(7)}$	$\mathfrak{e}_{8(8)}$	*dim* 3

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$\mathbb{C}_s$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{R}) \times \mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(6,\mathbb{R})$	$\mathfrak{e}_{6(6)}$	*dim* 5
$\mathbb{H}_s$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{sl}(6,\mathbb{R})$	$\mathfrak{so}(6,6)$	$\mathfrak{e}_{7(7)}$	*dim* 4
$\mathbb{O}_s$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(6)}$	$\mathfrak{e}_{7(7)}$	$\mathfrak{e}_{8(8)}$	*dim* 3

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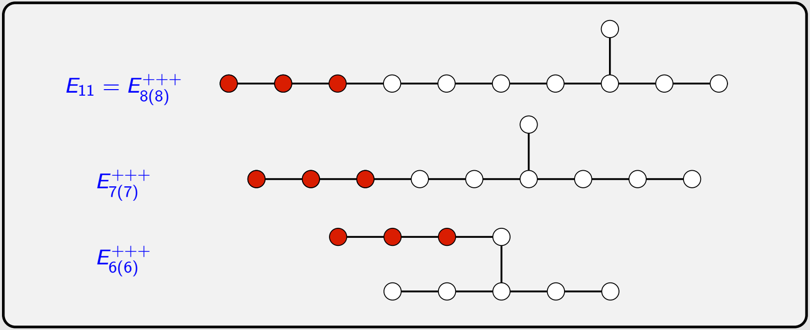
$\mathbb{R}$	$\mathfrak{so}(3)$	$\mathfrak{sl}(3, \mathbb{R})$	$\mathfrak{sp}(6, \mathbb{R})$	$\mathfrak{f}_{4(4)}$	
$\mathbb{C}_s$	$\mathfrak{sl}(3, \mathbb{R})$	$\mathfrak{sl}(3, \mathbb{R}) \times \mathfrak{sl}(3, \mathbb{R})$	$\mathfrak{sl}(6, \mathbb{R})$	$\mathfrak{e}_{6(6)}$	*dim* 5
$\mathbb{H}_s$	$\mathfrak{sp}(6, \mathbb{R})$	$\mathfrak{sl}(6, \mathbb{R})$	$\mathfrak{so}(6, 6)$	$\mathfrak{e}_{7(7)}$	*dim* 4
$\mathbb{O}_s$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(6)}$	$\mathfrak{e}_{7(7)}$	$\mathfrak{e}_{8(8)}$	*dim* 3

Non−supersymmetric theories

Uplift



3	4	5	6	7	8	9	10
E_8	E_7	E_6	$SO(5,5)$	$SL(5)$	$SL(3) \times SL(2)$	$GL(2)$	$SL(2)$
E_7	$SO(6,6)$	$SL(6)$	$SL(4) \times SL(2)$	$SL(3) \times U(1)$	A $SL(2) \times U(1)$ B $SL(3)$	$U(1)$	1
E_6	$SL(6)$	$SL(3) \times SL(3)$	$SL(2) \times SL(2) \times U(1)$	$SL(2) \times U(1)$	$SL(2)$		

The $E_{7(7)}$ theory

Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
10	–				**1**				
9	$\mathbb{R}^+$	**1**		**1**	**1**		**1**	**1**	
8A	$GL(2,\mathbb{R})$	**2**	**1**	**2**	**1**	**2**	**3** **1**	$2\times\mathbf{2}$	**3** $2\times\mathbf{1}$
8B	$SL(3,\mathbb{R})$		**3**		$\overline{\mathbf{3}}$		**8**		**15** **3**
7	$GL(3,\mathbb{R})$	**3** **1**	**3**	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$ **1**	**8** **1**	**8** $\overline{\mathbf{6}}$ **3**	**15** $\overline{\mathbf{6}}$ $2\times\mathbf{3}$	
6	$SL(4,\mathbb{R})$ $\times$ $SL(2,\mathbb{R})$	$(\mathbf{4},\mathbf{2})$	$(\mathbf{6},\mathbf{1})$	$(\overline{\mathbf{4}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\overline{\mathbf{20}},\mathbf{2})$ $(\mathbf{4},\mathbf{2})$	$(\mathbf{64},\mathbf{1})$ $(\overline{\mathbf{10}},\mathbf{1})$ $(\mathbf{6},\mathbf{3})$ $2\times(\mathbf{6},\mathbf{1})$		
5	$SL(6,\mathbb{R})$	**15**	$\overline{\mathbf{15}}$	**35**	$\overline{\mathbf{105}}$ **21**	$\overline{\mathbf{384}}$ **105** $\overline{\mathbf{15}}$			
4	$SO(6,6)$	**32**	**66**	**352**	**2079** **462** **66**				
3	$E_{7(7)}$	**1**	**1539** **1**	**40755** **1539** **1**					

The $E_{7(7)}$ theory

Maximal Supergravity									
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
10 IIB	$SL(2,\mathbb{R})$		$\mathbf{2}$		$\mathbf{1}$		$\mathbf{2}$		$\mathbf{3}$
9	$GL(2,\mathbb{R})$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$(\bar{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\bar{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $(\bar{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{3},\mathbf{3})$ $2\times(\mathbf{3},\mathbf{1})$

Non Susy Magic $E_{7(7)}$									
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
10	–				$\mathbf{1}$				
9	$\mathbb{R}^+$	$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$	
8A	$GL(2,\mathbb{R})$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$ $\mathbf{1}$	$2\times\mathbf{2}$	$\mathbf{3}$ $2\times\mathbf{1}$
8B	$SL(3,\mathbb{R})$		$\mathbf{3}$		$\bar{\mathbf{3}}$		$\mathbf{8}$		$\mathbf{15}$ $\mathbf{3}$

The $E_{7(7)}$ theory

Maximal Supergravity									
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
10 IIB	$SL(2,\mathbb{R})$		$\mathbf{2}$		$\mathbf{1}$		$\mathbf{2}$		$\mathbf{3}$
9	$GL(2,\mathbb{R})$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\bar{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\bar{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $(\bar{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{3},\mathbf{3})$ $2\times(\mathbf{3},\mathbf{1})$

Non Susy Magic $E_{7(7)}$									
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
10	$-$				$\mathbf{1}$				
9	$\mathbb{R}^+$	$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$	
8A	$GL(2,\mathbb{R})$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$ $\mathbf{1}$	$2\times\mathbf{2}$	$\mathbf{3}$ $2\times\mathbf{1}$
8B	$SL(3,\mathbb{R})$		$\mathbf{3}$		$\bar{\mathbf{3}}$		$\mathbf{8}$		$\mathbf{15}$ $\mathbf{3}$

Singlet Truncation

$$G_{maximal} \supset G \times SL(2,\mathbb{R})$$

The $E_{7(7)}$ theory

Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
10	–				**1**				
9	$\mathbb{R}^+$	**1**		**1**	**1**		**1**	**1**	
8A	$GL(2,\mathbb{R})$	**2**	**1**	**2**	**1**	**2**	**3** **1**	$2\times\mathbf{2}$	**3** $2\times\mathbf{1}$
8B	$SL(3,\mathbb{R})$		**3**		$\overline{\mathbf{3}}$		**8**		**15** **3**
7	$GL(3,\mathbb{R})$	**3** **1**	**3**	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$ **1**	**8** **1**	**8** $\overline{\mathbf{6}}$ **3**	**15** $\overline{\mathbf{6}}$ $2\times\mathbf{3}$	
6	$SL(4,\mathbb{R})$ $\times$ $SL(2,\mathbb{R})$	$(\mathbf{4},\mathbf{2})$	$(\mathbf{6},\mathbf{1})$	$(\overline{\mathbf{4}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\overline{\mathbf{20}},\mathbf{2})$ $(\mathbf{4},\mathbf{2})$	$(\mathbf{64},\mathbf{1})$ $(\overline{\mathbf{10}},\mathbf{1})$ $(\mathbf{6},\mathbf{3})$ $2\times(\mathbf{6},\mathbf{1})$		
5	$SL(6,\mathbb{R})$	**15**	$\overline{\mathbf{15}}$	**35**	$\overline{\mathbf{105}}$ **21**	$\overline{\mathbf{384}}$ **105** $\overline{\mathbf{15}}$			
4	$SO(6,6)$	**32**	**66**	**352**	**2079** **462** **66**				
3	$E_{7(7)}$	**1**	**1539** **1**	**40755** **1539** **1**					

The $E_{6(6)}$ Theory

Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
8	$SL(2,\mathbb{R})$			$\mathbf{2}$			$\mathbf{3}$	
7	$GL(2,\mathbb{R})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$	$\mathbf{4}$ $2\times\mathbf{2}$
6	$SL(2,\mathbb{R})$ $\times$ $SL(2,\mathbb{R})\times\mathbb{R}^+$	$(\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{2})$	$(\mathbf{2},\mathbf{2})$	$(\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$ $(\mathbf{1},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$ $(\mathbf{2},\mathbf{3})$ $(\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{2})$	$(\mathbf{4},\mathbf{2})$ $(\mathbf{2},\mathbf{4})$ $3\times(\mathbf{2},\mathbf{2})$ $(\mathbf{3},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	
5	$SL(3,\mathbb{R})$ $\times$ $SL(3,\mathbb{R})$	$(\mathbf{3},\mathbf{3})$	$(\overline{\mathbf{3}},\overline{\mathbf{3}})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{8})$	$(\overline{\mathbf{6}},\mathbf{3})$ $(\mathbf{3},\mathbf{3})$ $(\mathbf{3},\overline{\mathbf{6}})$	$(\overline{\mathbf{15}},\overline{\mathbf{3}})$ $(\overline{\mathbf{3}},\overline{\mathbf{15}})$ $2\times(\overline{\mathbf{3}},\overline{\mathbf{3}})$ $(\mathbf{6},\overline{\mathbf{3}})$ $(\overline{\mathbf{3}},\mathbf{6})$		
4	$SL(6,\mathbb{R})$	$\mathbf{20}$	$\mathbf{35}$	$\mathbf{70}$ $\overline{\mathbf{70}}$	$\mathbf{280}$ $\overline{\mathbf{280}}$ $\mathbf{189}$			
3	$E_{6(6)}$	$\mathbf{78}$	$\mathbf{650}$ $\mathbf{1}$	$\overline{\mathbf{5824}}$ $\mathbf{5824}$ $\mathbf{650}$ 78				

The $E_{6(6)}$ Theory

Maximal Supergravity									
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{3},\mathbf{3})$ $2\times(\mathbf{3},\mathbf{1})$
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}$ $\overline{\mathbf{15}}$	$\mathbf{70}$ $\mathbf{45}$ $\mathbf{5}$	

Non Susy Magic $E_{6(6)}$								
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
8	$SL(2,\mathbb{R})$			$\mathbf{2}$			$\mathbf{3}$	
7	$GL(2,\mathbb{R})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$	$\mathbf{4}$ $2\times\mathbf{2}$

The $E_{6(6)}$ Theory

Maximal Supergravity									
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{3},\mathbf{3})$ $2\times(\mathbf{3},\mathbf{1})$
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}$ $\overline{\mathbf{15}}$	$\mathbf{70}$ $\mathbf{45}$ $\mathbf{5}$	

Non Susy Magic $E_{6(6)}$								
Dim	Symmetry	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
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7	$GL(2,\mathbb{R})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$	$\mathbf{4}$ $2\times\mathbf{2}$

Singlet Truncation

$$G_{maximal} \supset G \times SL(3,\mathbb{R})$$

The $E_{6(6)}$ Theory

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8	$SL(2,\mathbb{R})$			$\mathbf{2}$			$\mathbf{3}$	
7	$GL(2,\mathbb{R})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$	$\mathbf{4}$ $2\times\mathbf{2}$
6	$SL(2,\mathbb{R})$ $\times$ $SL(2,\mathbb{R})\times\mathbb{R}^+$	$(\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{2})$	$(\mathbf{2},\mathbf{2})$	$(\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$ $(\mathbf{1},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$ $(\mathbf{2},\mathbf{3})$ $(\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{2})$	$(\mathbf{4},\mathbf{2})$ $(\mathbf{2},\mathbf{4})$ $3\times(\mathbf{2},\mathbf{2})$ $(\mathbf{3},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	
5	$SL(3,\mathbb{R})$ $\times$ $SL(3,\mathbb{R})$	$(\mathbf{3},\mathbf{3})$	$(\overline{\mathbf{3}},\overline{\mathbf{3}})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{8})$	$(\overline{\mathbf{6}},\mathbf{3})$ $(\mathbf{3},\mathbf{3})$ $(\mathbf{3},\overline{\mathbf{6}})$	$(\overline{\mathbf{15}},\overline{\mathbf{3}})$ $(\overline{\mathbf{3}},\overline{\mathbf{15}})$ $2\times(\overline{\mathbf{3}},\overline{\mathbf{3}})$ $(\mathbf{6},\overline{\mathbf{3}})$ $(\overline{\mathbf{3}},\mathbf{6})$		
4	$SL(6,\mathbb{R})$	$\mathbf{20}$	$\mathbf{35}$	$\mathbf{70}$ $\overline{\mathbf{70}}$	$\mathbf{280}$ $\overline{\mathbf{280}}$ $\mathbf{189}$			
3	$E_{6(6)}$	$\mathbf{78}$	$\mathbf{650}$ $\mathbf{1}$	$\overline{\mathbf{5824}}$ $\mathbf{5824}$ $\mathbf{650}$ 78				

Ehlers Truncation

The Non supersymmetric theories appearing in the doubly split magic square are $SL(2,\mathbb{R}) - Singlet$ or $SL(3,\mathbb{R}) - Singlet$ truncation of the maximal theory

$\mathbb{A} \setminus \mathbb{B}$	$\mathbb{R}$	$\mathbb{C}_s$	$\mathbb{H}_s$	$\mathbb{O}_s$
$\mathbb{R}$	$\mathfrak{so}(3)$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{f}_{4(4)}$
$\mathbb{C}_s$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{R}) \times \mathfrak{sl}(3,\mathbb{R}) \times \mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(6,\mathbb{R}) \times \mathfrak{sl}(2,\mathbb{R})$	$\mathfrak{e}_{6(6)}$
$\mathbb{H}_s$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{sl}(6,\mathbb{R}) \times \mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{so}(6,6) \times \mathfrak{sl}(2,\mathbb{R})$	$\mathfrak{e}_{7(7)}$
$\mathbb{O}_s$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(6)} \times \mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{e}_{7(7)} \times \mathfrak{sl}(2,\mathbb{R})$	$\mathfrak{e}_{8(8)}$

Ehlers Truncation and the Magic Triagle

The Non supersymmetric theories appearing in the doubly split magic square could be uplifted and the truncations of the maximal theories could be generalized to $SL(n,\mathbb{R}) - Singlet$ truncations; the result is the Magic Triangle

$SL(n) \setminus D$	3	4	5	6	7	8	9	10
	E_8	E_7	E_6	$SO(5,5)$	$SL(5)$	$SL(3)\times SL(2)$	$GL(2)$	$SL(2)$
$SL(2)$	E_7	$SO(6,6)$	$SL(6)$	$SL(4)\times SL(2)$	$SL(3)\times U(1)$	A $SL(2)\times U(1)$ B $SL(3)$	$U(1)$	1
$SL(3)$	E_6	$SL(6)$	$SL(3)\times SL(3)$	$SL(2)\times SL(2)\times U(1)$	$SL(2)\times U(1)$	$SL(2)$		
$SL(4)$	$SO(5,5)$	$SL(4)\times SL(2)$	$SL(2)\times SL(2)\times U(1)$	A $U(1)\times U(1)$ B $SL(2)\times SL(2)$	$U(1)$			
$SL(5)$	$SL(5)$	$SL(3)\times U(1)$	$SL(2)\times U(1)$	$U(1)$	1			
$SL(6)$	$SL(3)\times SL(2)$	A $SL(2)\times U(1)$ B $SL(3)$	$SL(2)$					
$SL(7)$	$SL(2)\times U(1)$	$U(1)$						
$SL(8)$	A $U(1)$ B $SL(2)$							
$SL(9)$	1							

$$G_D^{(n)} \longleftrightarrow G_{n+2}^{(D-2)}$$

Symmetry

$$G_D \supset G_D^{(n)} \times SL(n,\mathbb{R})$$

Ehlers

Non-Supersymmetric Magic Theories and Ehlers Truncations , *Alessio Marrani, Gianfranco Pradisi, Fabio Riccioni, Luca Romano* arXiv:1701.03031

- Non supersymmetric theories and the magic triangle
- Ehlers truncation
- Extension to non maximal cases
- U-duality orbits