Quantum corrections in AdS/dCFT

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Table of contents

Motivation

Conformal field theories:

- Phenomenologically relevant
- Highly constrain the form of correlation functions
- Success of understanding standard AdS/CFT setup and $\mathcal{N} = 4$ SYM theory, in particular due to integrability

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Conformal field theories:

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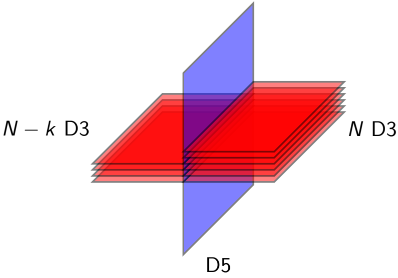
Defect CFTs:

- Equally relevant
- New features:
  - Non-vanishing one-point functions
  - Non-vanishing two-point functions between operators of different scaling dimensions
- New aspects of gauge gravity correspondence: AdS/dCFT

Table of contents

String-theory construction

D5-D3 probe brane set-up [Karch, Randall (2000)]

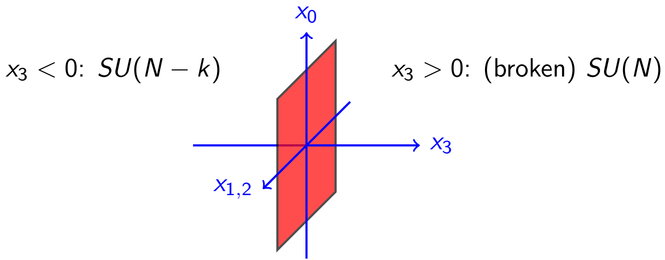


D3 brane $\sim \mathbb{R}^{1,3}$
D5 brane $\sim AdS_4 \times S^2$ with flux k through S^2

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	×	×	×	×						
D5	×	×	×		×	×	×			

(brace under x_0–x_2: defect)

Gauge theory



- $SU(N)$ broken by x_3-dependent vacuum expectation values for scalars
- 3D fundamental hypermultiplet on defect

$$S = S_{\mathcal{N}=4} + S_{D=3}$$

[DeWolfe, Freedman, Ooguri (2001)], [Erdmenger, Guralnik, Kirsch (2002)]

Classical solution

Classical fields

$$\phi^{\text{cl}}_{1,2,3} \neq 0 \qquad \phi^{\text{cl}}_{4,5,6} = 0 \qquad \psi^{\text{cl}} = \bar{\psi}^{\text{cl}} = 0 \qquad A^{\text{cl}}_{\mu} = 0$$

Equations of motion [Constable, Myers, Tafjord (1999)]

$$\frac{\partial^2}{\partial x_3^2}\phi^{\text{cl}}_i = [\phi^{\text{cl}}_j, [\phi^{\text{cl}}_j, \phi^{\text{cl}}_i]]$$

x_3: distance to defect

Solution via k-dimensional irreducible representation of the $SU(2)$ Lie algebra:

$$\phi^{\text{cl}}_i = -\frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{pmatrix}$$

t_1, t_2, t_3 with $[t_i, t_j] = i\epsilon_{ijk}t_k$

Also satisfies Nahm equation [Nahm (1979)]

Action

Action of $\mathcal{N}=4$ SYM theory

$$S_{\mathcal{N}=4} = \frac{2}{g_{\mathrm{YM}}^2}\int \mathrm{d}^4x\,\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\,\mathrm{D}_\mu\,\phi_i\,\mathrm{D}^\mu\,\phi_i + \frac{i}{2}\bar{\psi}\Gamma^\mu\,\mathrm{D}_\mu\,\psi + \frac{1}{2}\bar{\psi}\Gamma^i[\phi_i\,,\psi] + \frac{1}{4}[\phi_i\,,\phi_j][\phi_i\,,\phi_j]\Big]$$

Expand around classical solution

$$\phi_i = \phi_i^{\mathrm{cl}} + \tilde{\phi}_i \qquad i = 1,2,3$$

Gauge fix with $S_{\mathrm{gf}} = -\frac{1}{2}\,\mathrm{tr}(G^2)$, $G = \partial_\mu A^\mu + i[\tilde{\phi}_i\,,\phi_i^{\mathrm{cl}}]$

$$S_{\mathcal{N}=4} + S_{\mathrm{gf}} = S_{\mathrm{kin}} + S_{\mathrm{m}} + S_{\mathrm{cubic}} + S_{\mathrm{quartic}}$$

[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Mass terms

Mass term

$$S_{\mathrm{m}} = \frac{2}{g_{\mathrm{YM}}^2} \int \mathrm{d}^4 x \, \mathrm{tr} \Big[+ \frac{1}{2}[\phi_i^{\mathrm{cl}}, \tilde{\phi}_j][\phi_i^{\mathrm{cl}}, \tilde{\phi}_j] + \frac{1}{2}[\phi_i^{\mathrm{cl}}, \phi_j^{\mathrm{cl}}][\tilde{\phi}_i, \tilde{\phi}_j]$$
$$+ \frac{1}{2}[\phi_i^{\mathrm{cl}}, \tilde{\phi}_j][\tilde{\phi}_i, \phi_j^{\mathrm{cl}}] + \frac{1}{2}[\phi_i^{\mathrm{cl}}, \tilde{\phi}_i][\phi_j^{\mathrm{cl}}, \tilde{\phi}_j]$$
$$+ \frac{1}{2}[A_\mu, \phi_i^{\mathrm{cl}}][A^\mu, \phi_i^{\mathrm{cl}}] + 2i[A^\mu, \tilde{\phi}_i]\partial_\mu \phi_i^{\mathrm{cl}}$$
$$+ \frac{1}{2}\bar{\psi}\Gamma^i[\phi_i^{\mathrm{cl}}, \psi] - \bar{c}\,[\phi_i^{\mathrm{cl}}, [\phi_i^{\mathrm{cl}}, c]] \Big]$$

Properties:

- Non-diagonal in colour
- Mixing between the $\tilde{\phi}_1$, $\tilde{\phi}_2$, $\tilde{\phi}_3$ and A_3 as well as between the fermion flavours
- Mass proportional to $1/x_3$ via ϕ_i^{cl}

[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

How to solve this?

Diagonalising the mass matrix

Easy example: $A_0 = \begin{pmatrix} A_{0,k\times k} & A_{0,k\times(N-k)} \\ A_{0,(N-k)\times k} & A_{0,(N-k)\times(N-k)} \end{pmatrix}$

Mass term:

$$-\frac{1}{2x_3^2}\,\mathrm{tr}\left(A_0[t_i,[t_i,A^0]]\right) = -\frac{1}{2x_3^2}\,\mathrm{tr}\left(A_{0,k\times k}[t_i,[t_i,A^0_{k\times k}]]\right) + \frac{1}{x_3^2}\,\mathrm{tr}\Big(A_{0,k\times(N-k)}A^0_{(N-k)\times k}\underbrace{t_i t_i}_{\frac{k^2-1}{4}}\Big)$$

$L^2 = L_i L_i$ with $L_i = \mathrm{ad}_{t_i}$ is the Laplacian on the fuzzy sphere:

$\Rightarrow$ Can be diagonalised by fuzzy spherical harmonics $\hat{Y}_\ell^m$

Mass terms of $\{\tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3, A_3\}$ and the fermions also contain $\sigma_i L_i$

$\rightarrow$ Similar to spin-orbital interaction of the hydrogen atom!

[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Spectrum of the mass matrix

Eigenvalues (for $x_3 = 1$) and multiplicities in terms of $\nu = \sqrt{m^2 + \frac{1}{4}}$

Multiplicity	$\nu(\tilde{\phi}_{4,5,6}, A_{0,1,2}, c)$	$m(\psi_{1,2,3,4})$	$\nu(\tilde{\phi}_{1,2,3}, A_3)$
$\ell = 1, \ldots, k-1$	$\ell + \frac{1}{2}$	$\ell + 1$	$\ell + \frac{3}{2}$
$\ell + 1$	$\ell + \frac{1}{2}$	ℓ	$\ell - \frac{1}{2}$
$(k-1)(N-k)$	$\frac{k}{2}$	$\frac{k+1}{2}$	$\frac{k+2}{2}$
$(k+1)(N-k)$	$\frac{k}{2}$	$\frac{k-1}{2}$	$\frac{k-2}{2}$
$(N-k)(N-k)$	$\frac{1}{2}$	0	$\frac{1}{2}$

[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Propagators

Scalar propagator with x_3-dependent mass term

$$\left(-\partial_\mu \partial^\mu + \frac{m^2}{(x_3)^2}\right) K(x,y) = \frac{g_{\mathrm{YM}}^2}{2} \delta(x-y)$$

Standard scalar propagator $K_{\mathrm{AdS}}(x,y)$ in AdS_4 with mass $\tilde{m}$

$$(-\nabla_\mu \nabla^\mu + \tilde{m}^2) K_{\mathrm{AdS}}(x,y) = \frac{\delta(x-y)}{\sqrt{g}}$$

with the metric of AdS_4 given as $g_{\mu\nu} = \frac{1}{(x_3)^2} \eta_{\mu\nu}$

Scalar propagators

$$K(x,y) = \frac{g_{\mathrm{YM}}^2}{2} \frac{K_{\mathrm{AdS}}(x,y)}{x_3 y_3}$$

upon identifying $\tilde{m}^2 = m^2 - 2$

[Nagasaki, Tanida, Yamaguchi (2011)], [Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Table of contents

One-point functions in defect CFTs

New feature of dCFTs: operators $\mathcal{O}$ can have nonvanishing one-point functions [Cardy (1984)]

$$\langle \mathcal{O} \rangle = \frac{C}{x_3^\Delta}$$

Δ: scaling dimension of $\mathcal{O}$, x_3: distance to defect, C: constant

Studied in this dCFT at tree level for BPS operators [Nagasaki, Tanida, Yamaguchi (2011)] and operators in the $SU(2)$ sector [de Leeuw, Kristjansen, Zarembo (2015)], [Buhl-Mortensen, de Leeuw, Kristjansen, Zarembo (2015)], where integrability was found.

Study loop corrections $\rightarrow$ Start with simplest operator:

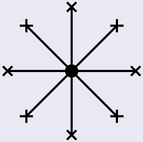
$$\mathcal{O}(x) = \mathrm{tr}(Z^L)(x)\,, \quad Z(x) = \phi_3(x) + i\phi_6(x)$$

BPS $\rightarrow$ corrections to C but not to Δ

One-point functions at tree level

Tree-level one-point function of $\mathcal{O} = \mathrm{tr}(Z^L)$ [Nagasaki, Tanida, Yamaguchi (2011)] [de Leeuw, Kristjansen, Zarembo (2015)]

$\langle \mathcal{O} \rangle_{\text{tree-level}} =$



$$= \mathrm{tr}((Z^{\mathrm{cl}})^L) = \mathrm{tr}((\phi_3^{\mathrm{cl}})^L) = \frac{(-1)^L}{x_3^L} \mathrm{tr}(t_3^L)$$

$$= \frac{(-1)^L}{x_3^L} \sum_{i=1}^{k} \left(\frac{k-2i+1}{2} \right)^L$$

$$= \begin{cases} 0\,, & L \text{ odd} \\ -\frac{2}{x_3^L (L+1)} B_{L+1}\left(\frac{1-k}{2}\right)\,, & L \text{ even} \end{cases}$$

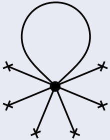
$B_{L+1}(u)$: Bernoulli polynomial

One-loop corrections to one-point functions

One-loop correction: two diagrams

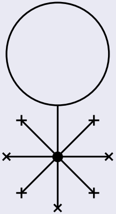
1. Two quantum fields in $\mathcal{O}$: tadpole diagram

$$\langle \mathcal{O} \rangle_{\text{1-loop,tad}} =$$



2. One quantum field in $\mathcal{O}$, one cubic vertex: lollipop diagram

$$\langle \mathcal{O} \rangle_{\text{1-loop,lol}} =$$



[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Tadpole diagram

Tadpole diagram

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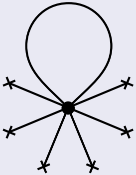


$$= \sum \operatorname{tr}(Z^{\text{cl}} \dots \tilde{Z} \dots \tilde{Z} \dots Z^{\text{cl}})$$

Planar limit $\rightarrow$ quantum fields need to be adjacent

Regulate scalar loop $K(x,x)$ in dimensional regularisation in the $d = 3 - 2\varepsilon$ dimensions parallel to the defect

Result:

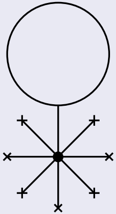
$$\langle \mathcal{O} \rangle_{\text{1-loop,tad}} = -\frac{\lambda}{16\pi^2} \frac{2L}{x_3^L (L-1)} B_{L-1}\left(\frac{1-k}{2}\right)$$

[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Lollipop diagram

Lollipop diagram

$$\langle \mathcal{O} \rangle_{\text{1-loop,lol}} =$$



$$= \sum \operatorname{tr}(Z^{\text{cl}} \dots \langle \tilde{Z} \rangle_{\text{1-loop}} \dots Z^{\text{cl}})$$

where

$$\langle \tilde{Z} \rangle_{\text{1-loop}}(x) = \tilde{Z}(x) \int \mathrm{d}^4 y \sum_{\Phi_1, \Phi_2, \Phi_3} V_3(\Phi_1, \Phi_2, \Phi_3)(y)$$

Result:

$$\langle \tilde{Z} \rangle_{\text{1-loop}} = 0 \qquad \Rightarrow \qquad \langle \mathcal{O} \rangle_{\text{1-loop,lol}} = 0$$

Crucially depends on the use of a supersymmetry-preserving regularisation scheme à la dimensional reduction!

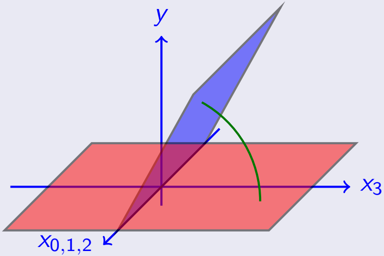
[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

String-theory calculation

Double-scaling limit suggested in [Nagasaki, Tanida, Yamaguchi (2011)] to compare gauge-theory and string-theory results and thus test AdS/dCFT:

$$N \to \infty \quad k \to \infty \quad k \ll N \quad \frac{\lambda}{k^2} \ll 1$$

Dual description of one-point function of $\mathcal{O}$ [Nagasaki, Yamaguchi (2012)]:



point-like string stretching from boundary of AdS_5 to D5 brane, calculable in supergravity approximation

Suggests perturbative expansion in $\frac{\lambda}{k^2}$

Comparison with string theory

String-theory result [Nagasaki, Yamaguchi (2012)]:

$$\left.\frac{\langle\mathcal{O}\rangle_{\text{1-loop}}}{\langle\mathcal{O}\rangle_{\text{tree-level}}}\right|_{\text{string}} = \frac{\lambda}{4\pi^2 k^2}\frac{L(L+1)}{L-1}$$

Gauge-theory result:

$$\left.\frac{\langle\mathcal{O}\rangle_{\text{1-loop}}}{\langle\mathcal{O}\rangle_{\text{tree-level}}}\right|_{\text{gauge}} = \frac{\lambda}{4\pi^2 k^2}\left(\frac{L(L+1)}{L-1} + O(k^{-2})\right)$$

Perfect match!

$\Rightarrow$ Non-trivial check of the gauge-gravity duality with partially broken supersymmetry and conformal symmetry

[Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]

Table of contents

Conclusions and outlook

Conclusions

- Initiated study of loop corrections in a class of dCFTs based on $\mathcal{N} = 4$ SYM theory, which have holographic duals involving background gauge fields with flux k
- Scalars in field theory have x_3-dependent vevs in the k-dimensional representation of $SU(2)$
  $\rightarrow$ x_3-dependent non-diagonal mass matrix
  $\rightarrow$ Diagonalised mass matrix and found standard AdS_4 propagators
- One-loop one-point functions of $\mathrm{tr}(Z^L)$
- Match string theory $\rightarrow$ highly non-trivial check of AdS/dCFT

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Further work based on our framework:

- Finite N for $\mathrm{tr}(Z^L)$, one-loop one-point functions in the $SU(2)$ sector [Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, MW (2016)]
- Infinite straight Wilson line $\rightarrow$ particle-interface potential [Nagasaki, Tanida, Yamaguchi (2011)],[de Leeuw, Ipsen, Kristjansen, MW (2016)]
- Circular Wilson loop [Aguilera-Damia, Correa, Giraldo-Rivera (2016)]

Conclusions and outlook

Outlook

- One-loop one-point functions in the $SU(2)$ sector $\rightarrow$ integrability
- Higher loops
- Bulk-boundary two-point functions $\rightarrow$ Related to one-point functions via conformal bootstrap [Liendo, Meneghelli (2016)]
- Bulk-bulk two-point functions $\rightarrow$ Nonvanishing for $\Delta_1 \neq \Delta_2$ $\rightarrow$ Generate CFT data via OPE
- Cusped Wilson loops $\rightarrow$ cusp anomalous dimension
- Polygonal Wilson loops $\rightarrow$ relation to amplitudes?
- Localisation?
- Yangian symmetry for smooth Wilson loops?
- Integrability for particle-interface potential as for quark-antiquark potential?
- ...

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Thank you!